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THE SPECTRUM OF HYDROGEN IN THE REGION OF EXTREMELY SHORT WAVE-LENGTH

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In a preliminary paper[1] the author has given the wave-lengths of more than 130 lines in the region of the spectrum lying between the values 1850 and 1030 tenth-meters. It is the object of the present paper to compare the results obtained by the author with those given by Schumann, to describe the apparatus used in this research, and to call attention to some new facts which have come to light since the publication of the first notice. The description has been made with some minuteness in the hope that an exact knowledge of the conditions necessary to success may prove of value to investigators who work in this field. Some attention has also been given to earlier and imperfect forms of the apparatus. For the author wishes, by flagging the pits into which he has fallen, to prevent other investigators from similar accidents.

The improvements over the method of Schumann which characterize this research consist in the introduction of a concave diffraction grating in place of fluorite prism and lenses, thus permitting the measurement of wave-lengths. The object of continuing the work has been to improve the accuracy of the measurements and to eliminate from the radiation obtained from a hydrogen tube those frequencies which were due to impurities.

[1] *Astrophysical Journal*, **19**, 263, 1904.

As it is unsafe to rely upon a process of extrapolation even with a grating spectrum, the two-slit method described in a previous paper[1] was employed. The spectroscope has been altered in construction to permit of all the adjustments required for this method, and finally the photographic plate itself has been bent to agree in curvature with the arc of the circle on which the spectrum is in theory formed. Very considerable increase in accuracy has thus been gained. The grating with which the work has been done possesses one extremely strong first spectrum; in fact, it is to its brilliancy that the success of the research is due. In spite of the feebleness of the other spectra, however, it has been found possible to obtain many of the stronger lines between λ 1550 and λ 1250 in the second spectrum. Their measurement therefore forms a valuable check on the numbers obtained by the two-slit method.

The elimination of the lines due to impurities from the spectrum of hydrogen necessitates the study of the spectrum of air. As has been set forth in the earlier paper, it is found most convenient to fill the spectroscope itself with pure hydrogen; in fact, if the lines of the shortest wave-length are to be obtained, the light-path must be entirely in this medium. No window between discharge tube and spectroscope is permissible. When, however, the spectrum of a gas other than hydrogen is to be studied, a window of fluorite must separate the discharge tube from the spectroscope. The extent of the spectrum is therefore limited by the transparency of colorless fluorite; and the absorption of this substance has formed a necessary part of this research. As a matter of fact, even fluorite of the best quality was found to absorb all light beyond wave-length λ 1200; the study of the spectra of gases other than hydrogen therefore terminates with this value.

In view of the fact that Schumann made use of two fluorite lenses and a fluorite prism, it seemed extremely probable that his spectrum does not extend beyond wave-length λ 1200. To test the matter, the plates published in the *Smithsonian Contributions to Knowledge*, No. 1413, have been compared with the normal spectrum obtained during this research, and it has been found possible to identify a great majority of the hydrogen lines in this prism spectrum with lines measured

[1] *Physical Review*, **16**, 257, 1903.

by the author. Two important results follow. First, a scale of wave-lengths has been attached to the Schumann spectrum, as shown in the half-tone reproductions at the end of this memoir. Second, since the line of lowest wave-length visible in Schumann's plates has the value λ 1267, the present limit, λ 1030, establishes a considerable extension of the spectrum.

Since the effect of change in the electrical conditions under which a spectrum is produced is extremely important, the question of the existence of a secondary spectrum of hydrogen in the region of short wave-lengths has been examined. No such spectrum appears to exist; that is to say, there is but one hydrogen spectrum between λ 2000 and λ 1200.

The following pages contain a detailed account of the work of which the foregoing paragraphs may serve as an outline.

THE SPECTROSCOPE

The apparatus consists of two parts, the spectroscope itself and the vacuum receiver in which it is inclosed. The spectroscope is formed of a drawn brass tube 9.1 cm in internal diameter, 96 cm long, and 1.5 mm thick, one end of which is provided with an arrangement for holding the grating, while the other end carries the plate-holder and slits. The grating mounting consists of a square brass plate pivoted to turn about a vertical axis. The grating is held against this plate by springs, while screws through the back of the plate permit of the necessary adjustment about a horizontal axis.

At the end not occupied by the grating a draw-tube fits into the large tube. Upon the end of this draw-tube are mounted the slits and plate-holder in a manner shown in Figs. 1, 2, and 3, which may be described as follows: A circular brass disk closes the end of the draw-tube and is pivoted about the points *A A* (Fig. 3). The motion of this disk is regulated by the screws *X X*. Upon the disk are mounted the two slits *S S*. The width of the slits is controlled by the usual screw adjustment. In order to be able to adjust the slits parallel to each other, one of them is mounted in a tube which turns in the disk, the amount of this twist being regulated by the lever *L*.

The plate-holder *C* is so constructed that several photographs may be taken without withdrawing it from the apparatus. To this end the

disk carries two ways, *D D*, in which the plate-holder slides. The position of the holder in the ways is controlled by the lever *E*, pivoted about the point *F*. One end of this lever carries the pin *G*, while the other end is provided with an iron armature *H*. The pin *G* engages

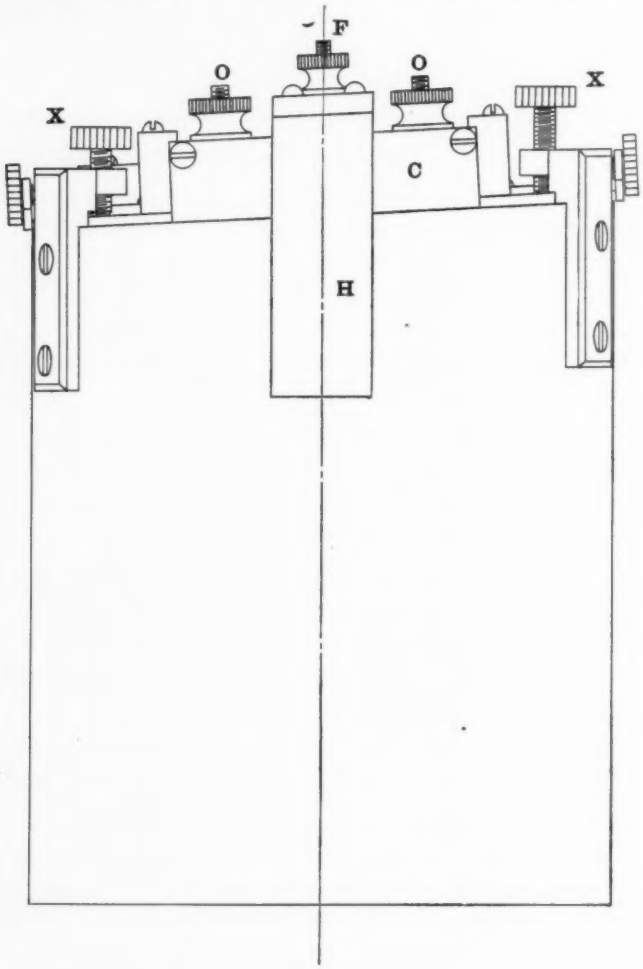


FIG. 1

one of the horizontal rods, *I*, and thus holds the plate-holder in position. To shift this position it is only necessary to swing the lever about *F* by means of a magnet exterior to the apparatus; the pin *G* then slips past one of the rods, *I*, and the plate-holder falls by an

amount corresponding to the distance between two rods. The plate-holder is designed to permit the plate to be bent to the arc of a circle of given curvature. To this end it is constructed in two parts, the outside case *C* and the movable form *M*. The form (shown with-

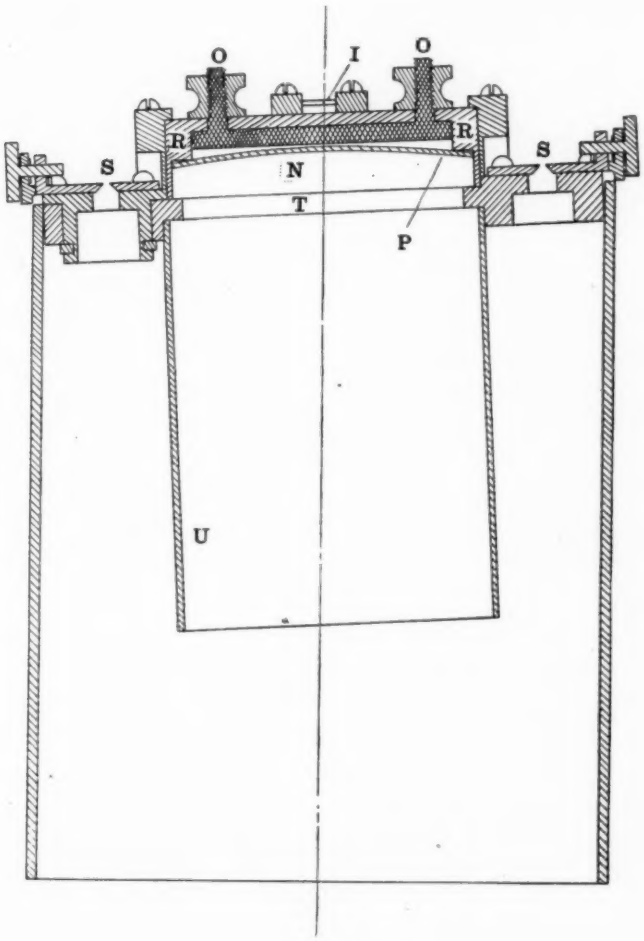


FIG. 2

drawn from the case, Fig. 4) carries two strips, *N N*, whose under sides are cut to the desired curvature; the ends of these strips project beyond the main body of the form. The plate *P* is slipped into the form and is tangent, when unbent, to the curved strips at their middle

point. The form is then drawn into the case by means of the screws *O O*, the ends of the plate come up against the shoulders *R R*, and as the screws are tightened the plate is bent to coincide with the strips *N N*.

The apparatus is so constructed that the curve to which the plate is bent passes through the slits. Light has access to the plate through

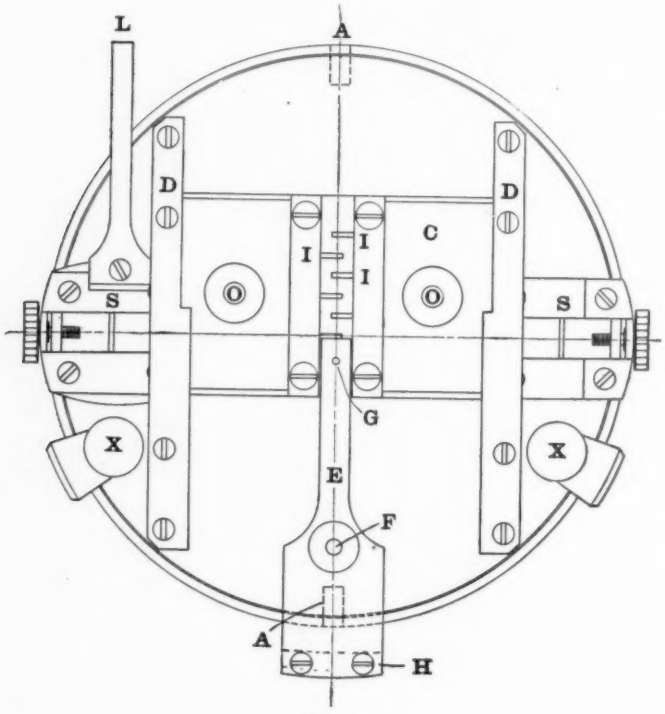


FIG. 3

a slot *T* cut in the disk, which slot also serves as a diaphragm for the spectrum. A sleeve, *U*, shields the plate from scattered light; and to reduce the reflection from the walls of the tube a set of circular diaphragms is provided. The whole system, draw-tube and large tube, is blackened inside by the usual process. In the early work it was proposed to inclose the spectroscope as above described in a large glass tube, but, owing to the difficulty of closing such a receiver air-tight, and owing to the great liability of tubes of this size to break, the

plan was abandoned. The receiver at present in use consists of a drawn brass tube 11.3 cm in diameter, 110 cm long and 1.8 mm thick. It is provided with two flanges, one at each end, cut from sheet brass and soldered to the tube. The flange at the end destined to be nearest the grating is closed by a circular brass plate, ground true, some 17 cm in diameter. Plates of two kinds have been used to close the other end of the receiver. In the simple form shown in Fig. 5, a circular brass disk was pierced only by the two holes destined to admit light to the slits of the spectroscope. In the more complex form, Fig. 7, a hand-hole is also provided through which the plate-holder may be introduced. This hole is 6.2 cm in diameter and is closed air-tight by a conical plug. In order to give this plug a sufficient bearing,

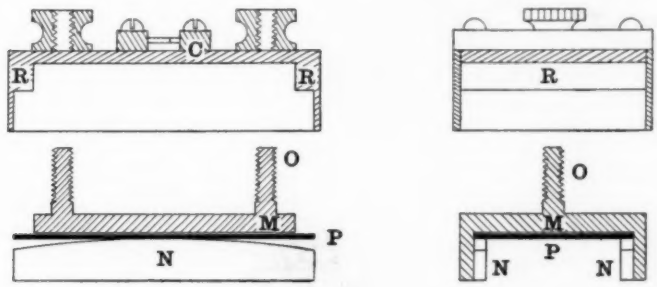


FIG. 4

a sleeve some 4.5 cm high is attached to the face plate. An inlet tube inserted about midway down the length of the receiver serves to exhaust the air; a wooden frame holds the apparatus horizontal. To facilitate the handling and development of the dry plates, the end of the receiver is inserted in a small dark-room. Fig. 6, shows the appearance of this arrangement. Into the receiver thus described the spectroscope is slipped, and small hard-rubber legs hold it in a central position. Fig. 8 shows the end of the apparatus with the face-plate removed.

The concave grating with which the work has been done was ruled in 1903 on the improved engine at Johns Hopkins University. The material is the usual speculum metal, the radius 97 cm, there are 15,028 lines to the inch. The diamond point was selected with the object of throwing as much of the light as possible into one spectrum.

To the great success which attended this effort the results of the work are due, for the instrument possesses one first spectrum of extreme brilliancy.

As the experiment is carried on in an atmosphere of hydrogen, the preparation of the gas forms an important factor. Zinc and hydrochloric acid of the greatest commercial purity obtainable are used. The gas is passed over potassium hydrate and collected over distilled water. Before the gas is admitted to the spectroscope it is dried over calcium chloride and phosphorous pentoxide. The drying tubes are protected at each end by a stopcock; thus the gas does not flow through the system directly, but stands over the material for some minutes before entering the spectroscope. The perfect dryness of the gas is necessary for the success of the work. All connections between hydrogen apparatus, tubes, and spectroscope are of glass. The evacuation is effected by a Geryk oil-pump driven by an electric motor, and the pressure is read by a McLeod gauge properly protected by drying-tubes. Here again all connections are of glass. All air admitted to the spectroscope is passed through a separate set of drying-tubes. This precaution has been found necessary to prevent the appearance of absorption bands. The joint between the brass receiver and the system of glass tubing is effected by a glass sleeve made tight with De Khotinski cement. Though this form of joint leaves something to be desired, nothing better has as yet been devised.

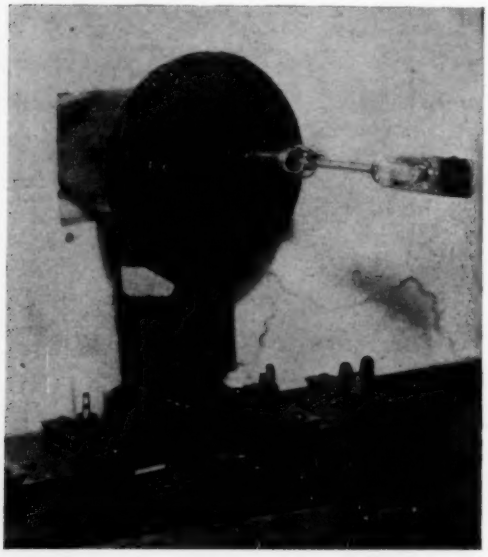


FIG. 5

The use of a discharge tube separated from the receiver by a fluorite window necessitates a separate pumping system, for the tube

must be exhausted apart from the receiver and filled with the gas to be studied. For this purpose a mercury pump by Kiss of Budapest has been used. The hydrogen is made electrolytically from a solution of barium hydrate and is dried over phosphorous pentoxide.

The form of the discharge tube depends upon the manner of making the experiment. If the tube is to communicate directly with the

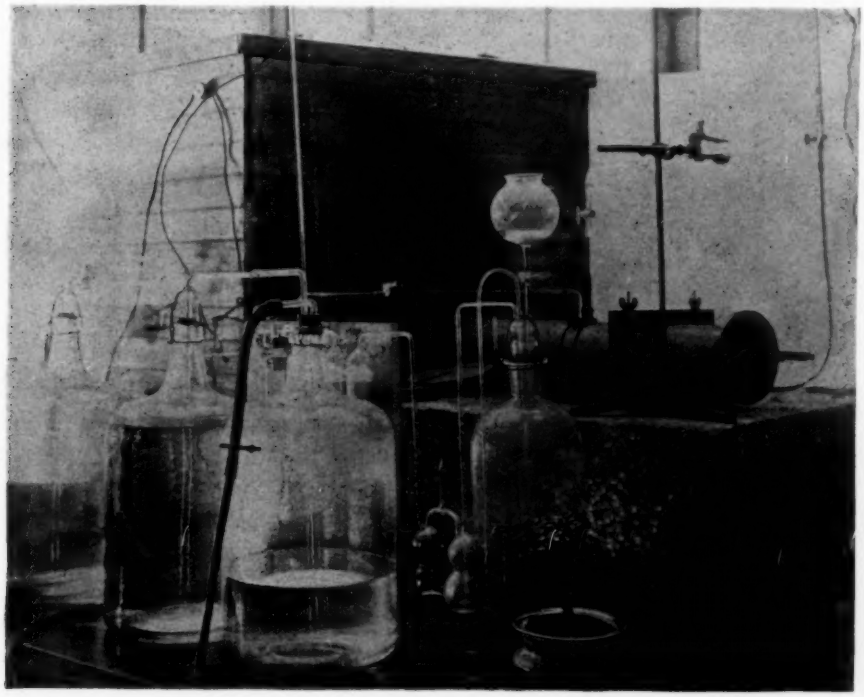


FIG. 6

receiver, so that the whole apparatus is filled from the receiver with hydrogen, the usual form of capillary tube with ring electrodes is employed. The dimensions in a typical case were as follows: length of capillary 6.4 cm, internal diameter 2.5 mm, diameter of electrodes 1.6 cm, distance of mouth of tube to electrode 4.5 cm. This last dimension is of special importance, since, if it be made too small, the discharge from the tube spreads into the receiver and produces fog, and if it be made too large, intensity of illumination is sacrificed.

If the tube is to be separated from the receiver by a window, and is to be separately exhausted, a special form is used (Fig. 9). Here the end of the internal capillary is brought as near the fluorite window as may be without undue heating. A device of this type not only brings the source of light near to the slit of the spectroscope, but reduces the absorption in the tube itself to a minimum. The last advantage is a most important one in dealing with gases such as air which absorb strongly. The electrodes in both forms of tube were usually of aluminium, but iron and copper have also been tried.

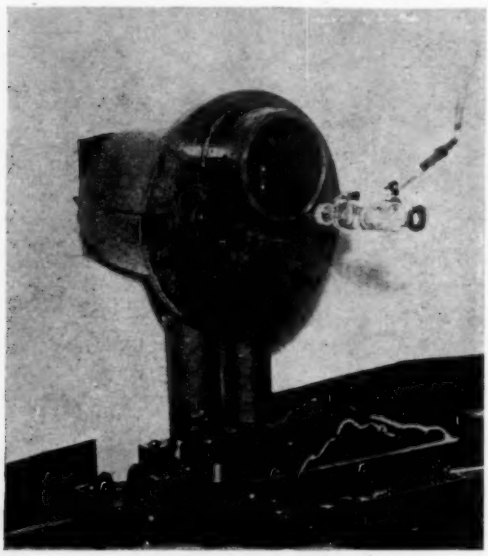


FIG. 7

ADJUSTMENT

After the spectroscope is placed in the receiver, the grating is turned until that part of the first spectrum to be investigated falls on the photographic plate. The arrangement of two slits serves a double purpose, as by it either the method of shifted spectra or the second spectrum comparison method may be used without altering the position of the grating. For no matter which method is to be employed, the grating is so placed that light from the right-hand slit gives the region of short wave-length in the first spectrum, while by illuminating the left-hand slit a shifted first spectrum is obtained superposed upon a shifted second spectrum. The dimensions of the apparatus are such that when the longest wave-length which falls on the plate from the ight-hand slit lies in the region of λ 1900, the longest wave-length in the shifted first spectrum has a value of about λ 3100. Observation of lines in the shifted spectrum serves therefore as a simple test of the exact position of the grating. When this position has once been reached, the grating end of the receiver is closed, a very little vaseline

being used in the joint, and the edge is luted with shellac or De Khotinski cement of the softer kind. It next becomes necessary to prepare the other face-plate. If the shifted spectrum method is to be employed this process consists in covering that hole which is to admit light to the left-hand slit with a quartz window, and to seal the discharge-tube over the right-hand opening. This last adjustment is a tedious one, for the mouth of the discharge-tube must be ground at such an angle that the capillary lies in the line determined by the slit and the grating center. This can be done only by trial. When the correct angle has been arrived at, the tube is fastened to the face-plate with De Khotinski cement. To insure a strong joint, the brass surface must be heated during the operation. The face-plate with the tube thus attached is rubbed evenly with a little white vaseline and applied to the flange of the receiver. Here great care must of course be used that the tube is in line with the slit. To facilitate this operation, tubes of both forms are made double-ended, that is, they have a quartz window by means of which it is possible to look through the capillary to the slit and thus assure correct alignment. Once in position, the plate is clamped and the edge luted with cement. Fig. 7, illustrates the appearance of the more improved form of plate and discharge-tube in position.

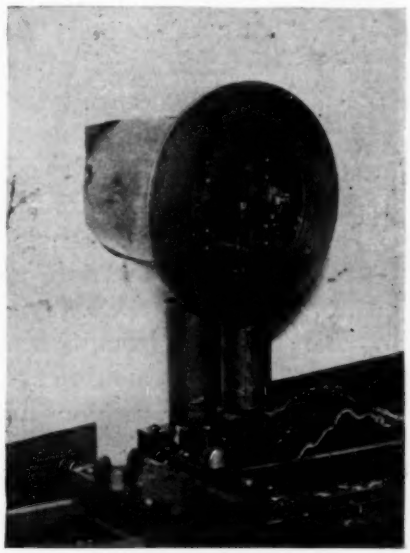


FIG. 8

The fact that the end of the receiver is in a dark closet permits the plate-holder to be placed in the ways of the spectroscope through the hand-hole without danger of fog. The hand-hole is next closed by the conical plug, and around the edge of the joint a little shellac is spread. The apparatus is now ready to exhaust. If no window is used between discharge-tube and spectroscope, both parts of the ap-

paratus are of course exhausted together and both are filled with hydrogen together. If a window separates the two, the tube must be exhausted by the mercury pump and filled from the separate supply of hydrogen. In either case the most laborious part of the adjustment lies still ahead; for the spectra from both slits must be in focus at the same time, and the position of the plate-holder can be determined only by trial. It is therefore necessary to take a series of spectrograms, removing the face-plate after every trial in order to change the adjustment of the spectroscope, and replacing the plate on each occasion air-tight in order to exhaust and fill with hydrogen. As can be easily understood from the figure, in order that the conditions of adjustment should be fulfilled, both slits must lie on the circle whose diameter is the grating's radius of curvature, and the plate must form a part of the arc of this circle. By construction the curve to which the plate is bent passes through the slits. There are then two degrees of freedom of adjustment, the draw-tube can be run in and out, and the disk can be turned about the axis $A\,A$; these two motions will suffice to bring the slits and plate into their correct theoretical positions.

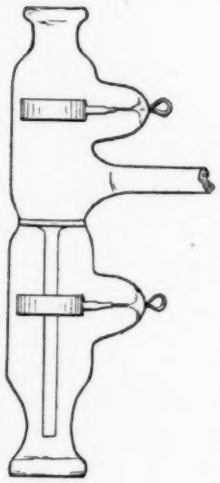


FIG. 9

Tedious as is the method of trial above described, it has seemed better to adopt it rather than to complicate the apparatus by the introduction of devices to regulate the focus from outside the receiver. Such devices might permit the focus to be changed without admitting the air, but the author is not at present prepared to face the problem of moving joints which must be maintained air-tight. Once the spectroscope is in adjustment, the face-plate, if it is of the improved form, can remain permanently in place.

As regards pumping the apparatus, and as to the extent to which it is necessary to wash with hydrogen with a direct connected discharge-tube, the following example may be of interest. The receiver and drying-tubes were first exhausted to 0.7 mm of mercury. The tubes were then shut off and filled with hydrogen; after the gas had stood over the drying material for two or three minutes, it was admitted to

the receiver. A second filling of hydrogen was let into the drying-tubes and in turn run into the receiver. The tubes are of such a capacity that two fillings raise the pressure by about 15 cm of mercury. The pump was then applied and the pressure reduced to 0.45 mm; hydrogen was admitted, the pump again applied until the pressure again reached about 0.45 mm, and an equal amount of hydrogen was for the third time admitted. It was usual to repeat this process of washing at least four times before a photograph was tried.

In making the exposure the end-on tube was excited by the transformer described in the previous paper. For the best results that pressure was chosen which gave a brilliant discharge in the tube without being so low as to permit the glow to spread from the tube into the spectroscope. The best value for the pressure under the conditions was in the neighborhood of 1.5 mm. The receiver was generally pumped to this pressure before the tube was excited.

In a plate such as that previously published where several spectra appear upon one negative, it was usual to allow a fresh supply of hydrogen to enter the receiver between each exposure. Thus after the plate-holder has been lowered by the magnetic device the receiver is re-pumped. It is just to suppose that the gas in the apparatus is much purer during the last exposure than during the first. The effect of this increased purity upon the nature of the spectra themselves has been noted in the former article; unfortunately the reproduction did not show the effect at all well, though it was extremely clear upon the original negative.

The process of washing, pumping, and rewashing is of necessity a tedious one and generally occupied the better part of a day. Schumann has observed that the appearance of the hydrogen spectrum in its visible part was no criterion of its purity as observed in the region of short wave-length. It may be of interest to add, nevertheless, that the discharge-tube when properly washed with hydrogen showed the many-line spectrum of that gas in a state of considerable purity. The appearance of air lines was always a sure warning, if a discharge-tube without a window was used, that the spectrum on the photographic plate would be extremely feeble.

In work of this kind it is found almost impossible to make the receiver absolutely air-tight. In fact, some of the most successful

of the early plates were obtained in the presence of a slight leak. Under the circumstances the magnitude of this leak becomes of importance. For example, the plate of the previous article was obtained with the surprisingly large leak of 0.2 mm in an hour, showing that perfect tightness of the apparatus was not necessary when proper attention was given to washing with hydrogen. If, however, the receiver was to be exhausted below 0.1 mm, the leak must be very much reduced. Practice and care in setting the face plate secured this result, and work has been done where the leak in twenty-four hours was less than 0.02 mm.

In those cases where the discharge tube is separated from the spectroscope by a fluorite window what has been said about the purity of the gas in the receiver and the amount of washing necessary to secure it remains of course true. To this is now added the trouble of pumping and filling the discharge-tube with whatever gas may be under examination. The gas in the discharge-tube, however, may be used over a considerable range of pressure, for the presence of the fluorite window prevents the discharge from spreading into the spectroscope. In practice the pressures varied in different experiments from 2 mm to 0.5 mm. The pressure in the receiver was usually reduced to 0.1 mm, though, if the hydrogen be pure, so low a pressure is not at all necessary. The width of slit used varied from 0.09 mm in the case of the crude plate published in a former article, to about 0.025 mm in the case of the fine plates from which measurements have been made.

The time of exposure for the hydrogen spectrum varies between five and thirty minutes, according to the width of slit and the sensitiveness of the plate.

DRY PLATES

Little can be added to Schumann's description[1] of the manufacture of the special dry plates necessary in this work. The first part of the research was carried on with plates prepared from glass 1.5 to to 2 mm in thickness. When the form of the spectroscope was improved and it became necessary to bend the plates to the arc of a circle, special sheets of thin glass were required. In order that the emulsion may flow evenly the plates must be very flat. This necessity of flatness, together with the mechanical difficulties of grinding,

[1] *Annalen der Physik*, **5**, 349, 1901.

put a limit to the thinness of the sheets. In practice plates 8.8×13 cm and between 0.4 and 0.5 mm thick are flowed, and when dry are cut into small pieces 2.6×4.4 cm.

One slight departure from the method of Schumann has been found advisable: each plate was separately supported on legs during the process of flowing. In this way, if the emulsion runs over the edge of one plate, only that plate is spoiled; while if all the plates are on one leveling table, a disaster to one may result in the overflow of all.

In development the author has used ortol with good results. Here, as has been remarked by Schumann, the strength of the developer must be regulated by the age of the plate. The addition of ice is a very necessary part of the process. The following proportions are suitable to plates six months to a year in age: Ortol A, 1 part; B, 2 parts; water, 2 parts; ice, about 1 part.[1]

ELECTRIC APPARATUS

The electric apparatus used to excite the discharge-tube has in almost all cases consisted of a transformer run from the 60 cycle 110-volt alternating circuit and provided with a suitable rheostat in the primary. When such a transformer is used with a discharge-tube containing gas at pressures from 1 to 0.1 mm, the addition of capacity across the terminals of the tube produces—with most gases—very little effect on the nature of the discharge, because the low resistance of the tube, after the current has once begun to pass, does not permit the condensers to charge. If a spark-gap be introduced in series with the tube, this difficulty is of course obviated. In all the earlier work no gap was used, so the spectra obtained were due to a discharge practically without capacity. The capacity when introduced consists of glass plates coated with tin-foil and has a value of perhaps 0.005 microfarads. In some of the work a coil with a mechanical break taking 12 volts and 5 amperes in the primary has been substituted for the transformer. In the case of the metallic spectra used for comparison the spark has of course been brightened by the use of capacity.

[1] A. Water . . . . . 1000 c.c.
Metabisulphate of potash 7.5 gr.
Ortol . . . . . 15 gr.

B. Water . . . . . 2000 c.c.
Pot. carb. . . . . 120 gr.
Sod. sulphite . . . 360 gr.
Hypo 1 to 20 . . . 20 c.c.

METHOD OF TESTING FLUORITE

In order to provide a window of the greatest possible transparency for the discharge-tube in those cases where the spectra of gases other than hydrogen were to be examined, it was necessary to test various specimens of fluorite which the author had at his disposal. The method was as follows: The piece under trial was attached to the plate-holder at the end of an arm in such a way that it projected to the right of the ways in which the holder moves. The length and shape of the arm were so adjusted that when the plate-holder was at its highest position the fluorite was just above the right-hand slit; but when the holder had been allowed to fall, the fluorite slab fell with it and came between the slit and the mouth of the discharge tube. The receiver was exhausted and filled with hydrogen in the usual way, and a rather wide slit was used. A photograph was then taken with the plate-holder at its highest position, thus the light path lay entirely in hydrogen. Next, by means of the magnetic device, the plate-holder was allowed to fall until the specimen of fluorite came in front of the slit; the light from the tube now passed through the fluorite before reaching the slit. By comparing the two spectra obtained one below the other on the same plate the point in the spectrum at which the specimen cut off the light could be easily determined.

Six circular plates of white fluorite 3 mm thick and 2½ cm in diameter, and two plates 2 mm thick—all from Zeiss of Jena—have been tested, with the result that, while none of them is absolutely opaque to light above λ 1600, their transparency varies very much. In no case, however, was any line of wave-length shorter than λ 1200 obtained, and of the eight pieces but two showed this transparency. The abrupt nature of the absorption at this point is well shown by Spectra 2 and 3 in Plate XI. Spectrum 2 was taken with the internal capillary discharge-tube and fluorite window, and 3 with no window between tube and slit. The author is not, of course, prepared to say that no fluorite does exist transparent to light above λ 1200; he can only say that of the best specimens obtainable up to the present but two show even this limited transparency. The discovery of some substance transparent to light of the very shortest wave-length known to exist would be an important step. For our knowledge of the spectra of gases other than hydrogen is at present limited by the transparency of fluorite.

The effect of the thickness of the fluorite window was tested by taking a series of spectrograms through one of the two best specimens, and then reducing the thickness of the piece from 3 to 0.9 mm. A second series taken through this thinner window showed no extension of the spectrum whatsoever. This is a result which might have been expected from the work of Schumann, and which confirms, for this region, that slow increase of absorption with thickness which has been observed in other parts of the spectrum.

ABSORPTION OF THE AIR

The absorption of the air is the important factor in all investigations which have to do with radiations of short wave-length. Cornu was the first to investigate the matter systematically, but Schumann has vastly extended the work, and has given data on the relation of length of air-path to the limit of the spectrum. His method was to interpose between his source of light and the slit of his spectroscope a cell the thickness of which could be varied. This cell was filled with air at atmospheric pressure.

There is not much to add. The method here employed was as follows: The discharge-tube was separated from the spectroscope by a fluorite window, and spectrograms were taken with air in the receiver. Thus the light from the discharge-tube traversed a layer of fluorite, and then passed through air to the grating and back to the photographic plate—a distance of about 200 cm. By taking a series of spectrograms at different pressures the variation of the absorption with pressure could be observed. At the very beginning of the investigation the author was confronted by a puzzling and persistent phenomenon—the absorption of the air appeared to be selective, not total; for a broad absorption band appeared between λ 1790 and λ 1550, and remained undisturbed even when the pressure had been reduced to 0.17 mm. At this point the air permitted the remainder of the spectrum to pass nearly out to the limit of transparency of the fluorite window. It was only after the receiver had been frequently washed with carefully dried air that the absorption band disappeared. The phenomenon is therefore due to some impurity, possibly something which comes from the brass of which the receiver is made, and which only persistent pumping will remove.

It is not perfectly satisfactory to compare the values obtained by Schumann which are given in terms of the absorption of a column of air at atmospheric pressure with those obtained by the author. It may be of some interest to point out, however, that, if the lengths of two equivalent air-paths are to each other inversely as their corresponding pressures, the column of air in the receiver at 0.17 mm pressure 200 cm long is about equivalent to a column at atmospheric pressure 0.4 mm in length. Now, when the receiver was at this low pressure, light of wave-length a trifle below λ 1400 was recorded on the photographic plate. It appears therefore that a column of air 0.4 mm long will permit light of this short wave-length to pass in sufficient intensity to affect a photographic plate under the conditions of the experiment.

The expression of the absorption of the air in anything like an absolute system is a very difficult matter. The point of practical interest in this part of the research is the advantage of an atmosphere of hydrogen in the receiver. It is not easy to exhaust the apparatus to a sufficient degree of transparency, but by successive washings with hydrogen traces of air can be removed and its absorption very largely eliminated.

PURITY OF THE SPECTRUM

The spectra of hydrogen from which the wave-lengths recorded in the following tables were measured have been obtained under some variety of condition, but they all show considerable uniformity of appearance. The greatest difference occurs between those spectra obtained from an internal capillary discharge-tube closed by a window and those where the tube communicated directly with the receiver. Contrary to expectation, the spectra obtained under the latter condition are much purer than those which the first method yields. No matter with what care the closed discharge-tube is pumped and repeatedly washed with hydrogen, certain characteristic bands are bound to make their appearance to a greater or less degree. The nature of these bands is unfortunately only too clearly seen in Spectrum 2 of Plate XI. If, however, the tube communicates directly with the receiver and is filled with hydrogen along with it, these bands may be totally absent. Schumann has observed their presence and ascribes them to carbon monoxide. On this point the author cannot yet be

sure; certain it is, however, that they occur strongly in the spectrum of the air. (Compare Spectra 1 and 2, Plate XI.)

The means used to produce the hydrogen for the discharge-tube have been varied. Zinc and hydrochloric acid, and electrolytic action on both dilute sulphuric acid and on barium hydrate solution, have served as sources for the gas. Various shapes of discharge-tube have been tried, both closed and communicating directly with the receiver. Aluminium, copper, and iron have been used as the material of the electrodes, which in turn have been of various dimensions. The comparison of the plates taken under the above conditions serves as an excellent test of the true source of the radiations supposed to be due to hydrogen.

In addition, the spectra obtained by exciting the discharge-tube when filled with air at pressure between 2 and 0.5 mm have been compared with the spectra obtained when the same tube was filled with hydrogen. The lines found to be common to the two spectra have been eliminated as due to the air itself or to some impurity. Such a process may result in the loss of a few true hydrogen lines, but what remains can be safely attributed to that gas. Finally, this matter has been checked by a study of the behavior of suspected lines, as the purity of the hydrogen in the discharge-tube is increased. It must be remembered that the elimination of lines due to impurities by comparison of the air and hydrogen spectra can be applied only to those radiations which lie in that region for which fluorite is transparent. The results are to be found in the table of wave-lengths given at the end of this paper.

The general appearance of the spectrum may be described as follows: Between λ 2000 and λ 1675 the author can find no trace of radiation due to hydrogen, but he is not prepared to assert that a faint continuous spectrum may not exist. From λ 1675, however, the spectrum consists of a multitude of very fine lines with a maximum of intensity near λ 1600. Near λ 1300 something very like an absorption band occurs, due perhaps to some slight trace of impurity in the gas, but always present no matter under what conditions the gas may be produced or examined. Lines are visible in this band, but they are very feeble. The lines beyond the region limited by the absorption of fluorite are some of them as strong as any in the spectrum. The

shortest measured wave-length has the value λ 1030, but beyond this there are some very faint lines whose wave-length must be between λ 1000 and λ 1010. At present these lines form the limit of the spectrum.

The nitrogen-like appearance of the spectrum of air shown in Fig. 1, Plate XI, deserves attention. The bands are beautifully clear in the original negative, and their general character can be seen even in the reproduction.

EFFECT OF CAPACITY ON THE SPECTRA

The spectra both of air and hydrogen were obtained with no capacity in circuit with the discharge-tube beyond that afforded by the connections of the apparatus. The effect of capacity on the spectrum of both gases in the visible region is so striking, however, that it seemed worth while to study the phenomenon in this new region of short wave-length. Moreover, the recent attempts which have been made to extract from the change in spectrum with change in condition some evidence as to the nature of the vibrating system of electrons, make such experiments doubly interesting. For in this new region we are dealing with vibrations more than three times as rapid as those studied in the visible spectrum. This difference in rapidity might well be expected to differentiate the effect produced by a given change of condition on the visible spectrum from the effect produced by the same change on the region between λ 2000 and λ 1030. It is even possible to conceive that this differentiation might throw some light on the vibrating system itself.

The research is unfortunately beset with mechanical difficulties. Reference has already been made to the trouble experienced from the spreading of the discharge into the spectroscope and the resulting fog produced on the photographic plate. This difficulty is increased a hundred-fold if a disruptive discharge is sent through the tube, for in this case the whole interior of the spectroscope seems to become luminous, and a total fogging of the plate results. With great care as to regulation of pressure some spectrograms have been obtained, but they have never been perfectly satisfactory, since even if but a single spectrogram is taken on a plate, the time of exposure must be short. When the investigator turns from the direct-connected discharge-tube

to one closed from the receiver by a fluorite plate, he is confronted by a new difficulty. The fog indeed is prevented, but after a short time the violence of the disruptive discharge deposits a thin film on the fluorite window and renders it totally opaque. This film need be hardly noticeable by transmitted light, and yet it will be thick enough to absorb all wave-lengths beyond $\lambda 1800$. The material of the electrode exercises of course a pronounced influence, but even with aluminium, which shows the effect the least, the result is as above described. The annoyance of disconnecting the discharge-tube from its pump, removing the face-plate from the receiver, detaching the discharge-tube from the face-plate and cleaning the window, followed by the same set of operations in the inverse order, must be experienced to be thoroughly appreciated. When it is remembered that with a disruptive discharge this process must be gone through after about four exposures, the difficulty of this part of the research will be understood.

In practice the discharge-tube was filled with hydrogen, and a spectrogram taken without capacity; next a spark-gap was introduced in series with the tube, and capacity was put in parallel until the gas showed the four-line spectrum clearly. The appearance of the tube was constantly watched with a direct-vision spectroscope.

A similar set of experiments was tried for air. In both cases the material of the electrodes was altered in various experiments. It is important to observe that the nature of the electrode does not seem to affect the nature of the phenomena.

The effect of the introduction of capacity with hydrogen is to introduce five sets of new lines. These lines lie between $\lambda 1900$ and $\lambda 1400$; under favorable circumstances they are strong and characteristic. The appearance of the principal spectrum remains unaltered, except for a very slight weakening.

The effect of capacity on the spectrum of air is very different. The band spectrum is weakened to such an extent as to be almost wholly destroyed, and five sets of new lines are introduced. These new or secondary lines are identical with those which appear in hydrogen. Though some of these lines are always present both in hydrogen and in air with the disruptive discharge, they vary very much in intensity from experiment to experiment. This variation with the condition of the research, added to the fact that the secondary lines appear both in

hydrogen and in air, makes it almost certain that they owe their origin to some impurity common to both gases. The nature of this impurity can be decided only after the spectra of the other principal gases have been examined. At present it seems safe to state (1) that there is no secondary spectrum of hydrogen in the region above λ 2000; (2) that the introduction of capacity almost totally destroys even the primary spectrum of air; (3) that new and characteristic lines do come into existence, in both air and hydrogen, and that these lines are probably due to some impurity. In weighing the evidence here presented it must be remembered that these results have been checked by experiments performed under very varying conditions. The pressure and purity of the gases, the shape and character of the discharge-tube, the material of the electrodes, and the time of exposure are all factors which have undergone investigation.

METHODS OF MEASUREMENT

The methods used were two in number. The values of all the lines were first obtained by the two-slit method, and these values were then checked by obtaining the stronger lines in the second spectrum and comparing their positions with known iron lines in the first spectrum. For this last purpose the first and second spectra obtained from the left-hand slit were employed.

The two-slit method has been described elsewhere, but a brief account of its theory and its limitations may not be out of place here. If two slits, S and S', be placed on that circle whose diameter is the grating's radius of curvature, the illumination of these slits by white light will give rise to the images I and I' (Fig. 10). To each of these images a set of spectra will correspond. For the present purpose it is sufficient to concentrate the attention on the two first spectra. It is evident that these two spectra will be shifted with respect to each other by an amount depending on the distance between the slits. If a photographic plate be placed between S and S', and if the height of these slits be properly adjusted, one of these spectra will be superposed upon the other. At a given point, P on the plate, the light brought to focus from S will be of a shorter wave-length than that from S'. If the sources of light be so selected that wave-lengths in both spectra arriving at P have known values, then the shift of one spectrum with respect

to the other may be determined by comparison of these values. If the apparatus is in adjustment, both spectra are in focus upon the same circle, and the amount by which one spectrum is shifted over the other is a constant quantity; that is to say, if the shift is determined by comparing known lines at one end of the plate, it must have the same value at the other end. It is upon this property that success in the use of method depends.

It next becomes of importance to inquire to what extent small errors of adjustment will influence the constancy of the shift. Here the nature of the method upon which the observer must rely in determining the perfection of this adjustment must be remembered. The only practical test consists in the sharpness of focus of the two spectra. It is the object, then, so to manage matters that both spectra shall be in perfect focus throughout the plate's length and at the same time. The vital question at once suggests itself: Is this test sufficiently delicate for the present purpose? If very accurate results are required, the question must be answered in the negative. A little consideration makes it obvious that the relative position of the images I and I', and hence the shift, changes with the focus more rapidly than can usually be detected by the change in sharpness of the lines. In other words, if the shift were given, the proper focus could be accurately determined, but if the sharpness of focus must be relied on, then the true shift can be only approximately inferred. Or again, for practical purposes, the apparent shift varies slightly more rapidly with variation in adjustment than does the sharpness of the spectral lines. The foregoing is of course somewhat dependent on the manner in which the adjustment is made. In the apparatus in question the slits and the photographic plate are rigidly fixed on the arc of a circle. This arc is capable of being thrust in or out parallel to

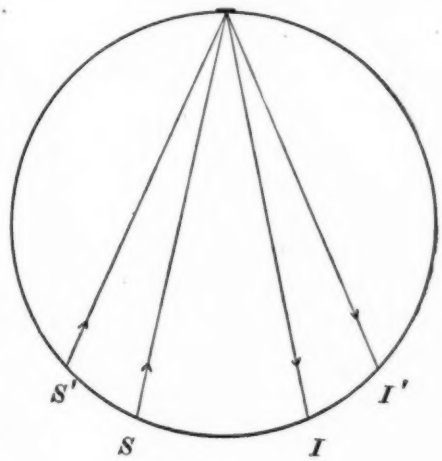


FIG. 10

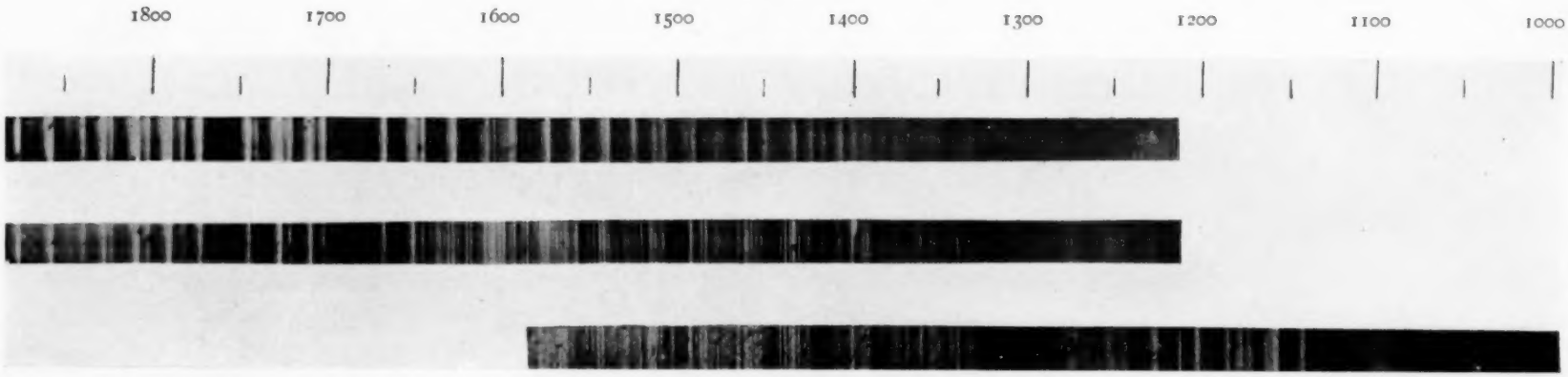
itself along a line connecting the center of the grating and the center of the photographic plate; it is also capable of rotation about its middle point. By these two movements perfect adjustment can be attained, but the test of this adjustment is not absolutely adequate.

The practical application of the method is as follows: The spectrum of iron was selected for comparison work. The grating was so turned that known lines in the spectrum of aluminium fell upon one end of the plate when the right-hand or direct slit was illuminated by light from a spark between terminals of the metal. The shift of the principal spectrum with respect to the comparison spectrum was then determined by comparing the positions of these lines in aluminium with known lines in the spectrum of iron. In order to insure accuracy, this shift determination was recorded on the same plate as the spectrum of hydrogen whose lines were to be measured. This was conveniently brought about by admitting the light from the aluminium spark directly through the discharge-tube, for which purpose the tube was fitted with a window of quartz at the end not attached to the face-plate. Upon the spectrum to be measured was superposed the comparison spectrum of iron, and in this spectrum fiducial lines were selected. The relative value of these lines was then obtained by subtracting the shift from their real value, previously corrected to vacuum. These relative values were then used as points of departure to determine the wave-lengths of the unknown gas spectrum. In practice the shift was 1180 Ångström units, so that the point in the iron spectrum falling on say $\lambda 1400$ of the gas spectrum has a value of $1400+1180=\lambda 2580$.

Owing to the dimensions of the plate a region of only about 760 tenth-meters can be photographed at one time. Thus if the aluminium line $\lambda 1935.29$ falls upon one extreme of the plate, the other end corresponds to wave-length $\lambda 1175$. In order to investigate light of shorter wave-length than this value, it is necessary to turn the grating, a process which necessitates a slight change in the adjustment of slits and plate.

To check the values obtained in the above manner, lines of short wave-length were obtained in the second spectrum. For this purpose the left-hand slit was covered by a discharge tube without a window, and the whole apparatus was filled with hydrogen exactly as usual.

PLATE XI



1. Air

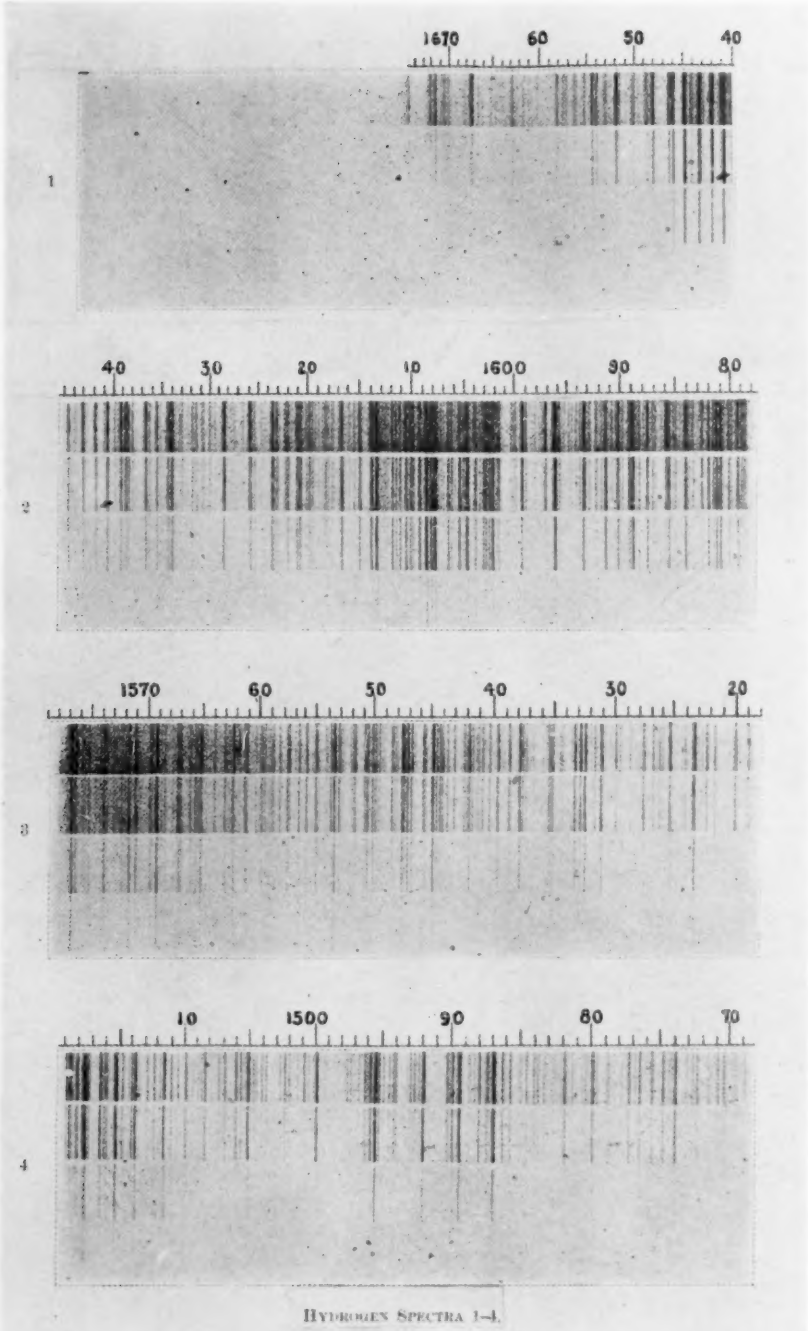
2. Hydrogen and Air

3. Hydrogen

ABSORPTION OF FLUORITE

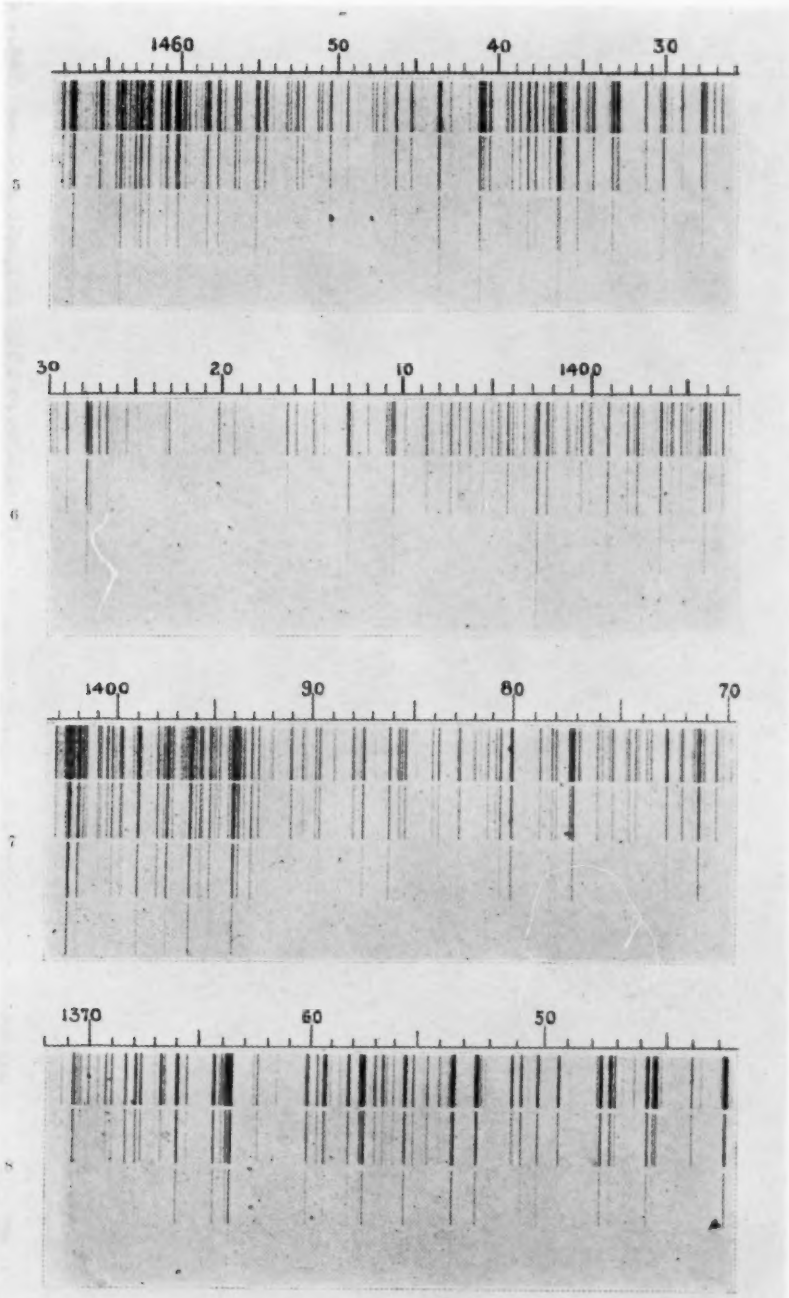
The two upper spectra were taken through fluorite

PLATE XII



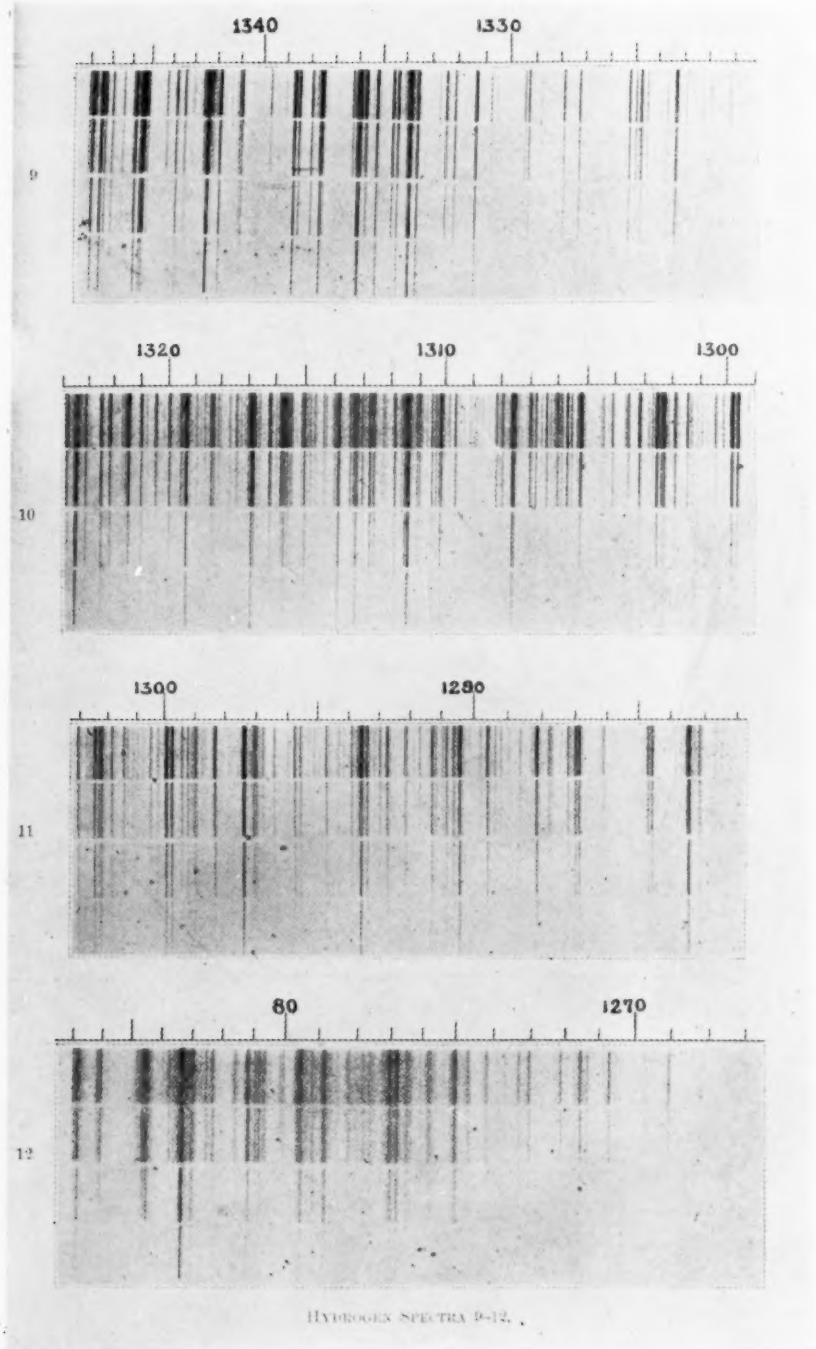
HYDROGEN SPECTRA 1–4.

PLATE XIII



HYDROGEN SPECTRA 5-8.

PLATE XIV



HYDROGEN SPECTRA 9-12.

Owing to the feeble character of the second spectrum, only the stronger lines between λ1550 and λ1250 could be photographed. Their position was determined by comparison with first-spectrum iron lines obtained from light which had passed directly through the discharge tube.

The plates have been measured on an engine made by Wolz of Bonn after the design by Kayser. The screw has been calibrated and proves to be of an accuracy far greater than this work demands. The intensities of the lines have been estimated, first, by observations made under the reading microscope, and, second, by projecting the spectrum on a screen. The latter method has the advantage that the whole spectrum is before the observer at one time. The values were estimated from plates taken without a fluorite window. The tables are divided into two parts. In the first are given 310 lines lying in that region from which it has been possible to eliminate the lines due to impurities. The error should not be greater than 0.3 Ångström units. In the second are lines in that region beyond the transparency of fluorite; their origin is not absolutely known, but they are probably due to hydrogen, since they were obtained when the discharge-tube was connected directly with the spectroscope, a condition under which air lines rarely occur. The error in these values should not be greater than one unit. The values of the iron lines are from the measurements of Exner and Haschek as given in Watts' tables;[1] the correction to vacuum come from the same source. The wave-lengths of the aluminium lines are from the measurements of Eder and Valenta.[2] The agreement between the tables and the numbers given in the "Preliminary Measurements" is well within the accuracy claimed for the earlier values.

SCHUMANN'S SPECTRUM

In order to compare the prismatic spectrum obtained by Schumann with the values of the table, the twelve plates published in the Smithsonian Memoir[3] were cut out and pasted together. The resulting spectrum, some 127 cm long, was placed upon a movable stand, and the grating spectrum was projected upon it by means of a lens. By changing the magnification so as to keep step with the dispersion, the

[1] W. M. Watts, *Index of Spectra*, Appendix J.

[2] *Beiträge zur Photochemie*, p. 388.

[3] *Smithsonian Contributions*, No. 1413.

strong lines in the one spectrum were identified with those in the other from λ 1674 to λ 1269 without the least difficulty. From the values thus obtained interpolation curves were drawn for each one of the twelve plates separately, and by means of these curves a scale of Ångström units was attached to each of the twelve illustrations. By the permission of Dr. Schumann, and through the kindness of Professor Langley, of the Smithsonian Institution, the half-tone reproductions, Plates XII, XIII, XIV, which accompany this paper were then made from these illustrations. They by no means do justice to the fine originals, but, considering the difficulty of the process, they may be considered fairly satisfactory.

The agreement between the author's measured values and the prismatic lines is extremely gratifying. Of the 285 lines given in the tables, all but three or four are found in Schumann's plates. There are, however, a considerable number of fainter lines in the prismatic spectrum not visible in the plates obtained with the grating. Moreover, owing to the fineness of the slit, and the great dispersion used by Schumann, some of the single lines of the table are seen, by comparison with the prismatic spectrum, to consist of doublets or triplets.

The excellent agreement between these two spectra obtained under such different conditions makes the existence of any chance impurity very improbable.

The extreme line in Schumann's map has the value λ 1266.9. That author has stated that he obtained some lines too faint to reproduce; from the angles given[1] it is difficult to calculate their exact wavelength, but it seems improbable that they should have a value much below λ 1230. In this connection it is interesting to note that the calculation of Martins[2] from the Ketteler-Helmholtz formula for fluorite was not far wrong.

LIMIT OF THE SPECTRUM

It may well be asked: To what is the present limit of the spectrum due? There are several causes which go to make up an answer to this question.

A much longer exposure might result in the discovery of new lines; unluckily there are difficulties in the way of this seemingly simple step; for, as has been previously stated, with a windowless tube there is a

[1] *Smithsonian Contributions*, No. 1413, p. 24.

[2] *Ann. der Physik*, **6**, 619, 1901.

great tendency for the discharge to spread into the receiver and cause fatal fogging of the plate. No plan has so far been devised to obviate this difficulty, and up to the present the length of exposure has been limited by it. Besides this mechanical difficulty, several other possible agents may exert an influence. Speculum metal may cease to reflect in the region near λ 1000; that it reflects so well as far as this point is surprising. The Schumann plates may cease to be sensitive. Small impurities in the hydrogen may exercise considerable absorption. Only experiments on metallic reflection, on the manufacture of plates, and on the purification of gases can answer these questions. The author sees no insurmountable difficulty, however, to the still further extension of the spectrum.

RESULTS

The results arrived at in this paper may be set forth as follows:

1. The spectrum has been extended from the limit obtained by Schumann to the value λ 1030.

2. The lines in the spectrum of hydrogen have been measured accurately from λ 2000 to λ 1228, and the values of the principal lines to λ 1030 have been determined.

3. The nature of the spectrum of air has been investigated.

4. The limit of transparency of certain specimens of white fluorite has been obtained.

5. The effect of the disruptive discharge on the spectra of hydrogen and air has been studied, and the absence of a secondary spectrum of hydrogen established in the region beyond λ 2000.

6. Wave-lengths have been attached to the spectrograms obtained by Schumann.

Much of this research has been carried on with the help of a grant from the Bache fund. The permission to reproduce the plates from the *Smithsonian Contributions* is due to the kindness of the secretary of that institution.

It is impossible to conclude this paper without some tribute to the man whose name will be always associated with the region of short wave-lengths which he discovered, and it is with the greatest pleasure that the author acknowledges the help and inspiration he has received from the friendship of Dr. Victor Schumann.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY,
December 27, 1905.

SPECTRUM OF HYDROGEN MEASURED BY A DIFFRACTION GRATING

Wave-Length	Intensity	Character	Wave-Length	Intensity	Character	Wave-Length	Intensity	Character
1228.3	8		1312.9	2		1386.3	3	
1230.1	8		1314.7	1		1387.7	4	
1231.0	1		1315.6	1	double	1390.0	1	double
1232.1	5		1319.2	4		1391.2	1	
1234.3	4		1323.4	5		1393.2	3	
1235.8	6		1325.0	5	double	1394.0	7	double
1239.6	3		1327.1	3		1395.2	2	
1241.5	2		1327.5	2		1396.4	7	
1246.1	4		1329.3	1		1397.5	6	
1247.2	4		1331.3	6		1398.0	1	
1248.0	2		1333.9	8	double	1399.0	7	
1249.8	3		1335.3	2		1400.6	1	
1251.2	3		1336.1	8	double	1402.0	4	
1253.2	6		1337.6	6		1402.8	8	
1253.9	5		1338.7	7	double	1404.3	5	
1255.5	4		1340.9	1	double	1405.5	2	
1257.1	4		1342.4	8		1407.3	7	
1258.2	4		1343.6	1		1408.6	3	
1259.9	4		1345.4	8	double	1410.5	8	triple
1261.9	8		1347.2	9	double	1411.8	1	
1264.0	1		1349.1	2		1413.0	8	
1264.6	5		1350.2	3		1414.9	2	
1265.8	4		1350.8	3		1416.4	3	
1267.3	1		1352.5	8		1419.5	2	
1268.3	1		1353.6	8		1420.3	3	
1269.1	3		1355.5	7		1426.8	3	
1269.9	1		1357.3	6		1427.8	7	double
1270.7	4		1358.2	4		1429.0	3	
1271.5	4		1359.2	5		1430.1	7	
1272.0	1		1360.1	5		1431.1	3	
1273.3	3		1362.4	1		1433.0	8	double
1274.2	1		1363.4	8		1434.3	3	
1275.0	3		1364.3	3		1435.2	4	
1276.1	1		1365.8	5		1436.3	7	double
1277.1	6	double	1366.5	1		1438.0	4	
1279.0	1		1367.6	3	double	1439.1	1	
1279.8	5		1368.0	3		1441.0	8	
1281.2	4		1369.1	1	double	1442.8	1	
1282.6	1		1370.4	2		1443.6	7	
1283.4	6		1371.3	6		1445.2	4	
1284.5	5	double	1372.1	1		1446.2	6	
1286.9	5	double	1372.9	3		1447.4	2	?
1288.1	4		1374.0	1		1449.2	2	
1289.4	3		1374.5	2		1450.3	5	
1290.4	5		1375.5	1		1451.0	1	
1291.3	2		1376.1	1		1452.0	3	
1293.6	6	double	1377.2	6	double	1452.5	1	
1295.7	2	double	1378.0	1	double	1454.3	1	
1297.4	5		1380.2	?		1455.1	7	double
1299.5	1		1380.8	1		1456.3	4	
1300.0	3	double	1382.9	1		1457.4	6	
1302.5	2	double	1383.0	1		1458.4	6	
1307.5	2		1384.2	1		1460.1	5	double
1311.1	2		1385.6	2		1461.0	4	

SPECTRUM OF HYDROGEN MEASURED BY A DIFRACTION GRATING—*Continued*

Wave-Length	Intensity	Character	Wave-Length	Intensity	Character	Wave-Length	Intensity	Character
1462.0	3		1535.0	6	double	1603.8	1	
1462.9	4		1536.7	1		1604.6	6	
1463.9	8		1537.5	7	double	1605.3	5	
1465.2	3		1539.2	5		1606.3	5	
1467.2	6	double	1539.9	2		1607.7	10	
1468.6	6		1540.6	2		1608.2	6	
1471.0	3		1541.6	7		1608.6	10	
1472.5	3		1543.9	2		1609.2	3	
1473.9	5		1544.7	8		1610.1	2	triple
1474.9	4		1545.5	2	double	1610.5	7	
1476.4	4		1546.4	6		1611.2	1	
1477.3	3		1547.4	7	double	1611.8	3	
1478.9	2		1548.3	1		1612.5	1	
1479.7	4		1549.9	7	double	1613.3	7	
1480.4	4		1550.6	7	double	1614.3	4	
1481.7	5		1551.5	2		1615.0	3	
1482.1	1		1553.3	10		1616.7	6	
1483.7	3		1554.9	3		1617.9	1	
1486.1	1		1555.6	1		1619.9	2	double
1486.9	9		1556.4	2		1621.1	7	double
1487.8	1		1557.4	1		1622.1	3	
1489.3	6		1558.7	1		1623.2	2	
1489.9	3		1560.0	1		1623.8	7	
1491.9	7		1561.1	2		1625.8	4	
1492.7	1		1562.2	4		1627.6	1	
1494.1	3		1563.0	1		1628.5	8	
1495.5	10	double	1564.0	1		1631.7	2	
1499.8	8		1565.1	3	double	1633.7	6	
1502.2	2		1567.1	5	double	1634.1	4	
1503.9	1		1569.2	6		1635.3	3	
1505.0	8		1569.7	1		1636.5	7	
1505.9	1		1571.3	1		1638.2	4	
1506.6	1		1571.7	7		1639.1	5	
1511.5	8		1574.3	5		1639.7	1	
1513.6	7		1577.2	8		1640.5	6	
1515.0	6		1579.2	4		1641.6	5	
1516.4	5		1581.0	7	double	1643.0	5	
1517.5	6	double	1584.1	7		1644.6	7	
1519.0	6		1585.7	7	double	1645.7	2	
1520.1	5		1587.6	3		1646.0	1	
1521.7	2		1589.0	8	triple	1647.8	1	
1522.5	2		1590.9	4		1651.8	1	
1523.4	8		1591.5	8		1654.2	2	
1525.4	5		1593.6	7		1662.9	1	
1526.6	2		1595.2	1		1667.4	2	
1527.5	4		1596.2	10		1670.2	1	
1529.7	3		1599.4	6		1671.6	2	
1530.9	6		1602.0	8		1672.4	2	
1532.1	6		1602.8	1		1674.6	1	
1533.2	6							

LINES OF UNCERTAIN ORIGIN PROBABLY DUE TO HYDROGEN

Wave-Length	Intensity	Character	Wave-Length	Intensity	Character	Wave-Length	Intensity	Character
1030.8	1		1148.8	2		1202.8	1	
1034.2	2		1151.2	2		1205.2	6	
1045.2	4		1160.9	10	double	1206.9	6	
1047.5	5		1164.0	6		1207.8	2	
1062.1	1		1166.5	6		1209.2	6	
1065.6	3		1169.2	1		1209.7	1	
1070.0	1		1172.6	1		1210.8	2	
1080.0	1		1174.9	1		1211.7	3	
1082.1	1		1176.2	5	double	1215.0	2	
1094.9	2		1178.5	3	double	1216.0	8	
1098.0	2		1180.8	7		1217.6	3	
1100.0	3		1182.7	4		1219.1	1	
1102.2	4		1185.0	2		1221.5	1	
1104.8	6		1189.0	7	double	1223.7	3	
1107.5	6		1198.6	2		1225.2	1	
1110.5	3		1200.2	2		1225.9	7	
1119.4	4		1201.8	3		1227.5	1	
1145.5	8	double						

THE RELATION BETWEEN THE SPECTRA OF SUN-SPOTS AND FOURTH-TYPE STARS

By WALTER M. MITCHELL

The recent investigations of Hale, Ellerman, and Parkhurst on the spectra of the stars of Secchi's fourth type have given a fresh impulse to the discussion of the question whether the peculiar spectrum of those stars is due to spots similar to those appearing on the Sun. A comparison is given[1] between the widened lines in the sun-spot spectrum and the lines in the spectra of fourth-type stars. The statistics of the widened lines are derived from the observations by Maunder during 1880, and those by Cortie during various periods prior to 1901. Their conclusion from the comparison is that the evidence favors strongly the view that spots like those on the Sun may form a characteristic feature of fourth-type stars.

Lockyer, by a somewhat similar comparison[2] between the spectra of third- and fourth-type stars and the widened lines observed in sun-spots at South Kensington, is led to the view that the fourth-type star is simply at a lower temperature, or at a more advanced stage of its life-history. The strong dark lines of its spectrum indicate stronger absorption, as in accordance with his theory of sun-spots the widened lines in their spectra are the results of absorption by cooler gas-masses.

On the other hand, Wilson supports the view that sun-spots are at a higher temperature than the surrounding photosphere, and, from the possible similarity of spot and fourth-type spectra, suggests that the fourth-type stars may be in reality hotter than those of preceding types.[3] The lack of brilliancy of the star is accounted for under the assumption that the temperature of the star is greater than that which permits the condensation of the vapors of certain elements to form a photosphere of the accepted type. It must be said, however, against this last statement that it is difficult to see how it would be possible for an intensely heated gas-mass to be exposed to the cold of space without suffering condensation somewhere in its outer layers.

[1] *Publications of the Yerkes Observatory*, **2**, 367.

[2] *Proc. R. S.*, **74**, 53, 1904. [3] *Monthly Notices*, **55**, 226, 1895.

The following table shows the results of a comparison of the sun-spot spectrum as observed at Princeton[1] with the lines in the spectrum of fourth-type stars recorded at the Yerkes Observatory,[2] and the lines in the spectrum of *α Orionis* observed by Keeler. The comparison is restricted to the region λλ 4861–6500.

EXPLANATION OF THE TABLE

The first column contains the wave-lengths of the lines in the sun-spot spectrum. Italics indicate that the line is of special importance, "R" that it has been seen reversed.

The second and third columns indicate respectively the probable origin of the lines as determined by Rowland, and the frequency of the line in the spot spectrum, estimated on a scale of ten.

The fourth column gives the conspicuousness of the line in the sun-spot spectrum, and indicates how greatly the line is affected. The scale ranges from 10 to −5; negative numbers indicate that the line is less conspicuous than in the photosphere. If discrepancies are noticed between the two latter columns and the corresponding columns in my table of spot lines, it is on account of subsequent observations which have been incorporated in this paper.

The fifth and sixth columns contain the wave-lengths and the intensities of the lines in the spectrum of fourth-type stars, as recorded by Hale, Ellerman, and Parkhurst.

The seventh and eighth columns give the same data for lines in the spectrum of *α Orionis*, intensity "E" indicating that the line is of equal strength with the corresponding line in the solar spectrum, "S" and "W" indicating that the line is respectively stronger or weaker than the corresponding solar line.

The ninth column is devoted to general remarks. A few bright lines in the red region of 152 *Schjellerup* are indicated by "(152)".

[1] *Astrophysical Journal*, **22**, 4, 1905.

[2] *Loc. cit.*

Affected in Sun-Spots				Fourth-Type Stars		Third-Type Stars		Remarks
Wave-Length	Origin	Frequency	Intens.	Wave-Length	Intens.	Wave-Length	Intens.	
F**4861.53	*H*	3	..	4861.3	10	4861.5	E	
4864.92	*V*	9	9	4865.1	2	4864.4	S	
4871.51	*Fe*	4	4	4871.5	2–3	4871.8	E	
4872.33	*Fe*	4	4					
4875.67	*V*	9	10	4875.5	2	4875.6	S	
4881.74	*V*	7	7	4881.6	3–4	4882.0	S	
4882.34	*Fe*	2	3					
4885.26	*Ti*	7	7	4886.0	4			Given by Hale as due to *Fe, V.*
4885.96	*Cr*	4	4					
4886.13	*Cr*	2	5					
4890.95	*Fe*	5	7	4890.3	1–2			Given by Hale as due to *V.*
4891.68	*Fe*	5	7					
4900.09	*Ti, La*	7	5	4900.1	2	4900.3	S	Given by Hale as due to *Ti, V.*
**4919.17	*Fe*	2	3			4919.2	E	
4920.68	*Fe*	2	4	4920.9	2–3	4920.7	E	
4921.15	*La*	2	2					
4921.96	*La, Ti*	6	7					
4925.75	*Ni*	2	−1	4925.2	1			Given by Hale as due to *Fe.*
4937.90	*Ti*	5	5			4937.5	S	
4957.48	*Fe*	3	4	4958.4	2	4957.6	E	
4957.79	*Fe*	3	4					
4958.43	*Ti*	2	5					
4981.91	*Ti*	3	4	4981.8	W			
4982.99		2	8					
5188.86	*Ti*	1	2	5189.2	1–2			
5191.63	*Fe*	2	3			5191.6	E	Reversed? in spot once.
*5195.11	*Fe*	1	4			5195		
*5202.52	*Fe*	4	5	5202.4	1			
*5204.77	*Fe–Cr*	5	5			5204.8	S	
*5205.90	*V*	1	−4	5205.8		5206.2	S	
*5206.22	*Cr–Ti*	5	5	to	10	5208.6	S	
*5208.60	*Cr*	4	5	5210.1		5210.6	S	
*5210.56	*Ti*	2	5					
5219.88	*Ti*	8	9			5219.6		Third-type line not in Sun?
**5226.71	*Ti*	2	..	5226.5	7			Weakened three times, reversed once in spot.
*5227.04	*Fe–Cr*	6	..					
*5233.12	*Fe*	3	5	5234.0	3			Given by Hale as due to *V.*
**5234.79		2	−3					
5238.74	*Ti*	9	8	5239.8	2			
5239.14	*Cr*	9	7					
5247.23 R	*Fe*	4	5	5247.4	3–4	5247.7	S	Reversed three times in spot.
*5247.74 R	*Cr*	7	5					Reversed five times in spot.
5250.39 *R*	*Fe*	10	9	5251.5	3–4	5250.6	S	Reversed seven times in spot.
5252.28	*Ti*	7	6			5252.3	S	

Affected in Sun-Spots				Fourth-Type Stars		Third-Type Stars		Remarks
Wave-Length	Origin	Frequency	Intens.	Wave-Length	Intens.	Wave-Length	Intens.	
5255.30 R	*Cr*	3	4	5255.6	1–2	5255.1	S	Given by Hale as due to *Fe, Cr.*
5255.49	*Mn*	5	6					
5265.32	*Cr*	2	4	5265.9	1			
5266.14	*Ti*	6	5					
E**5269.72	*Fe*	3	5	5270.4	4–5	5269.7	S	
						5270.4	S	
5296.87	*Cr*	6	6			5297.0	S	
5297.41	*Cr–Ti*	7	6					
5298.19	*Cr*	3	3	5298.2	3–4	5298.0	E	
5298.45	*Cr*	3	−2					
5298.67	*Ti*	3	−2					
5302.48	*Fe*	1	4	5302.5	1–2			
5307.54	*Fe*	1	3	5307.5	1			
5321.29	*Fe*	3	4	5320.8	2–3			
*5328.24	*Fe*	4	5	5329.0	3–4			
*5329.33	*Cr*	3	5					
5340.12	*Fe*	1	−2	5341.5	3	5341.2	S	
5341.34	*Fe, Mn*	2	−2			5346.0	S	
*5345.99	*Cr*	3	−1			5348.5	S	
5348.51	*Cr*	4	−2					
5349.65	*Ca*	4	2	5350.0	3			
5349.93	*Fe*	1	−2					
5366.83		4	5	5366.6	2	5370	E	
5369.78	*Co–Ti*	5	5					
**5371.70	*Fe–Cr?*	3	5	5371.7	7–8	5371.6	S	
5383.57	*Fe*	3	4	5384.7	1			
5396.78	*Ni*	4	6	5397.3	4			Given by Hale as due to *Fe, Ti.*
*5397.34	*Fe*	4	5					
5404.36	*Fe*	2	4			5404	W	
*5405.99	*Fe*	3	7	5406.4	1	5406	S	
5407.69	*Mn*	5	7	5408.3	2–3			
5409.02		4	8					
*5410.00	*Cr*	4	5	5410.4	2–3	5410.0	S	
5413.89	*Mn*	3	5	5414.2	1–2			
5420.55	*Mn*	7	6	5420.2	3			
5424.29	*Fe*	2	4			5424.2	E	
5426.47 R		10	10			5426.5		Reversed once in spot.
5429.35	*Ti?*	5	3	5430.2	3	5429.9	S	
**5429.91	*Fe*	3	4					
**5432.75 R	*Mn*	9	10			5433.0	S	Reversed three times in spot.
*5434.74	*Fe*	4	4	5434.3	1–2	5434.7	S	Given by Hale as due to *Fe, V.*
**5447.13	*Fe*	2	4	5447.8	7	5447.0	S	
5460.72		9	10	5460.9	1–2	5461.0		
5461.76 R		7	7					
5462.71 R	*Ni*	9	9			5463.0	W	Reversed six times in spot.
5466.61	*Fe*	4	5	5467.3	1–2			
5467.20	*Fe*	3	2					

Affected in Sun-Spots				Fourth-Type Stars		Third-Type Stars		Remarks
Wave-Length	Origin	Frequency	Intens.	Wave-Length	Intens.	Wave-Length	Intens.	
5474·44	*Ti*	6	7	5474·5	1–2			Given by Hale as due to *Fe*.
5477·90	*Ti*	7	7	5478·0	1–2	5477	E	
5483·31 R	*Fe*	3	4	5483·0	1–2			Reversed twice in spot.
5483·57	*Co*	6	5					
*5497·74	*Fe*	7	5	5498·0	4	5497·6	S	
*5501·68	*Fe*	4	6	5501·8	2	5501·6	S	
5506·09 R	*Mn*	6	8	5507·1	1	5507·0	S	
*5507·00	*Fe*	1	4					
5512·74	*Ti*	4	6	5512·4	2–3			
5513·20	*Ca*	3	5					
5525·77	*Fe*	2	5	5525·4	1–2			
5538·74 R	*Fe*	8	10	5539·5	7			Reversed seven times in spot.
5546·73 R	*Fe*	5	4	5546·6	1–2			
5547·22	*Fe—V*	6	5	5548·3	2			
5556·0		1	6	5556·4	1–2	Region here same as in Sun		Line not in photospheric spectrum.
5573·33		1	3	5573·7	1			
5583·1		1	4	5583·9	8			Line not in photospheric spectrum.
5584·53 R	*V*	6	4					
*5588·99	*Ca*	4	4	5589·2	1			
5594·69	*Ca*	3	4	5594·7	1–2			
5598·71	*Ca*	2	3			5598·5	S	
5619·82 R		5	5	5620·3	2–3			Reversed in spot twice.
5620·72 R	*Fe*	4	−3					Obliterated once, reversed twice.
5625·09		7	6	5625·1	4			Given by Hale as due to *Fe*, *V*.
5644·37	*Ti*	7	6	5644·2	1–2			
5657·66 R	*V?*	9	6	5658·2	1–2	5658·5	S	Reversed four times in spot.
**5658·10 R	*Y*	4	5					
5658·88	*Cr*	1	3					
5663·16	*Ti–Fe–Y*	5	4			5663·0	S	
5671·07	*V*	9	9	5671·3	1–2			
5672·05	*Sc*	9	8					
5708·32	*Fe*	5	3	5708·3	2–3			
5712·10	*Fe*	7	4	5712·8	1–2	5712	S	Always reversed in spot.
5712·36 R	*Fe*	8	5					
5712·99	*Cr*	3	3					
5727·27	*Ti–V*	7	8			5727·2	S	
5727·87 R	*Cr?–V?*	10	10					
5731·44 R	*V*	10	10	5731·6	3–4	5732·0	S	
5731·98	*Fe*	1	−2					
5743·18		2	5	5743·7	2			
5743·65	*V*	7	7					
5748·17 R	*Fe*	5	5	5749·4	2			Reversed five times in spot.
5748·57 R	*Ni*	8	9					

Affected in Sun-Spots				Fourth-Type Stars		Third-Type Stars		Remarks
Wave-Length	Origin	Frequency	Intens.	Wave-Length	Intens.	Wave-Length	Intens.	
5762.49	*Ti*	9	5	5762.5	2			
5778.68	*Fe*	2	3	5778.0	1			
5784.88 R	*Fe*	6	4	5784.8	3			Always reversed in spot.
5798.08		7	−2	5798.1	1			
5798.40	*Fe*	1	2					
5823.91		3	4	5822.9	2			
5848.34	*Fe*	4	3	5848.5	1			
5856.31	*Fe*	2	3			5856		
D_3**5875.98	*He*	3	3			5875		
D **5894.2	*Na*	5	5	5894.3	10	(??)		
5922.33	*Ti*	8	6	5921.9	1			
5944.95	*A(wv)*	3	5	5945.9	1			
6058.38		6	5	6059.1				
6085.47 R		8	7	6086.1				(152)
6098.46 R		3	3	6098.5	3			
6098.87	*Ti*	10	3					
6119.74	*V*	9	8	6119.0	2			
6129.19 R	*Ni*	10	4	6130.5	B 8			Reversed five times in spot. (152)
6131.79 R		1	4					
*6154.44 R	*Na*	10	8	6154.9	B 2			Reversed six times in spot. (152)
								(152)
6200.53 R	*Fe*	5	4	6200.9	B10			Given by Hale as due to *Ca, O.*
6269.08 R	*V*	10	8	6269.6	10			
6271.48	*Fe*	3	3					Reversed in spot seven times. (152)
6330.31 R	*Cr*	9	9	6330.2	B 1			
6355.25	*Fe*	3	5	6357.6	3			
6358.90 R	*Fe*	5	5					

A comparison of this kind is made with great difficulty on account of the very great difference in the dispersive powers employed in the two kinds of spectra. The spectra of the fourth-type stars were obtained in most instances with the three-prism spectrograph of the Yerkes Observatory, while for the sun-spot spectrum I have used the third order of a large Rowland grating, the ratio of the two resolving powers being about 1:14. To avoid this difficulty, if possible, the attempt was made to secure for the comparison the low dispersion spectrum of the umbra of a large sun-spot. This was done by substituting a single reflecting prism for the grating previously used, while the light from the spot umbra only was allowed to fall on the slit, the

remainder of the Sun's surface being properly screened off. The results were entirely negative: the brightness of the sky spectrum completely overpowered that from the spot, so that it was questionable, at first glance, whether the spot or a portion of the photosphere was on the slit. The appearance was simply that of the normal solar spectrum, with the exception that the Fraunhofer lines were of slightly greater intensity than usual. There was no indication whatever of bands or of bright regions.

The wave-lengths of the lines in the spectrum of *α Orionis* agree remarkably with those of the corresponding spot lines, but this is as would be expected owing to the greater brilliancy of this star, allowing greater dispersion and more accurate determination of wave-length.

There are many instances where several sun-spot lines have approximately the wave-lengths of the fourth-type lines, and in these cases it was found necessary to group the spot lines together, forming, as it were, one line. A number of fourth-type star lines agree closely in wave-length with strong Fraunhofer lines in the solar spectrum, while near the Fraunhofer lines are frequently faint lines which become very prominent in the spot spectrum, while the Fraunhofer lines are not affected, as instanced by the star line λ 5255.6, whose origin, as given by Hale, is *Cr, Fe.* The corresponding solar line is probably λ 5255.12, which is not affected in sun-spots, while the neighboring faint *Mn* line at λ 5255.49 is usually considerably widened. The star line at λ 5625.1 due to *Fe, V* has probably its counterpart in the solar line at λ 5624.77—*Fe, V.* This line I have never seen affected in the spot spectrum, but the near-by faint line at λ 5625.09 has been frequently widened and intensified. Still another instance is the star line λ 5731.6—*Fe, V*, the solar counterpart of which is probably the line λ 5731.98—*Fe.* This line has only once been observed in the spot spectrum, and then as "thinned," while near it the line at λ 5731.44 is one of the strongest spot lines. While it is to be noted that in most cases the wave-length of the star line agrees more closely with that of the spot line than with the wave-length of the stronger Fraunhofer line, it is to be remembered that the difference in wave-length in any case is considerably less than the limit of the resolving power of the Yerkes spectrograph in the given region, hence making it very uncertain whether the star line corresponds to the spot line or to the unaf-

fected Fraunhofer line. That the latter is the case seems to be borne out by the fact that there are many instances of agreement between strong Fraunhofer lines and star lines, while there are very few instances where faint lines in the solar spectrum which are much affected in the spot, have strong counterparts in the spectra of the fourth-type stars.

Again the star line may have for its counterpart a group of fine lines, only a few of which are affected in the spot; an instance is the star line λ 5298.2: in the solar spectrum there is a group of seven rather faint lines near this wave-length, only two of them being much affected in the spot spectrum. Another instance is the star line λ 5658.2, due to *V*, which falls in a group of six solar lines of varying intensities, only two of which are much affected in the spot spectrum.

Hence it will be seen that under the circumstances it is almost impossible to make a satisfactory comparison of the two spectra, and it tends to show that it is questionable whether we have any grounds for their supposed similarity. As it is impossible to tell from the wave-lengths of many of the star lines which is the corresponding solar line, the conclusions of other observers appear to have been based merely upon the apparent similarity of wave-lengths.

Now as to the intensities of the corresponding lines. It will be noted, on consulting the table, that even by giving the star lines the "benefit of the doubt" the agreement with the sun-spot spectrum is slight. The total number of star lines in the region λλ 4861–6500 is 143; in the table are given 90 which have approximately the same wave-lengths as lines in the spot spectrum, while 75 of these have their wave-lengths in fairly close agreement. But of the 75 there are only 14 lines (10 per cent. of the whole) which are prominent in both stellar and spot spectra. There are 16 lines which are prominent in the spot, while either faint or absent entirely from the star spectrum, and there are 5 which are the other way.

According to Hale,[1] the titanium lines show great similarity in the two spectra. He states: "No line which is missing in the star has an intensity greater than 1 in the arc, or a widening greater than 4 in sun-spots." With this I cannot agree, for the following lines are absent from the star, are strong in sun-spots, and have arc intensities

[1] *Loc. cit.*, 373.

indicated: λ 4913.8 (3), λ 5219.8 (2), λ 5471.4 (2) λ 5899.5 (3).[1] It may also be noted that the titanium lines λ 5336.97 and λ 5675.65 each have intensity 2 in the star and have not been observed in the spot spectrum at Princeton.

The magnesium lines, according to Hale, play a significant part in the fourth-type spectrum, the *b* group are a prominent feature, and other lines are present. It is to be noted that they are of small importance in the spot spectrum, no observations of magnesium lines being recorded at Princeton.

In contrast to this is the behavior of the manganese lines, which are of relatively great importance in the spot spectrum, and are apparently of minor consequence in the spectra of fourth-type stars.

Carbon, whose presence in the fourth-type stars is so strongly indicated by the absorption bands in their spectrum, is apparently absent from sun-spots.[2]

Hence, summing up the evidence, it is seen that of 90 possible coincident lines in the two spectra, only 14 are sufficiently intense in both spectra to warrant any conclusions as to the similarity of the sources of light. Likewise the non-agreement between the lines of magnesium, manganese, and carbon, with the irregularities in the behavior of the titanium lines, are facts against the similarity of the two spectra. Therefore from the Princeton sun-spot observations it seems improbable that spots such as exist on the Sun constitute a characteristic feature of the fourth-type stars.

PRINCETON, N. J.,
January 24, 1906.

[1] Hasselberg, *Astrophysical Journal*, **4**, 212, 1896.

[2] *Ibid.*, **22**, 6, 1905.

LINE STRUCTURE. II. THEORY OF BROADENING, DOUBLING, AND REVERSAL

By P. G. NUTTING

Considerable data relating to the structure and behavior of spectrum lines has accumulated during the last few years. Michelson and Fabry and Perot made investigations with the interferometers bearing their names. Recently James Barnes,[1] with a Fabry-Perot interferometer, has investigated the prominent lines of mercury, cadmium, thallium, zinc, and hydrogen. R. A. Houstoun[2] and Ludwig Janichi[3] have examined the same lines with echelon spectroscopes. The writer, also using an echelon, has recently investigated a great many of the visible lines of many of the elements, chiefly with a view to establishing types of variability in line structure. The present paper is a discussion of the results described in a previous paper.[4]

It was found that five types of behavior covered the observations, and that fully 95 per cent. of all lines observed were included in two classes, (2) and (3), being about equally divided between them. These five types are as follows: (1) lines like the chief mercury and helium lines, having components of the nature of distinct spectrum lines; (2) lines that simply broaden indefinitely as their intensity is increased, about half of which show superposed structure just before reversal; (3) lines that twin at an early stage with sharp, widely separated components; (4) lines that triple in the intermediate stage between single narrowness and reversal; (5) lines, few in number, that are unsymmetrical and vary irregularly. Attention is here chiefly directed toward the numerous lines of the second and third types. Lines of the first type are to be treated as miniature spectra. Regular tripling, type (4), appears to be a special case of twinning in which the central

[1] *Astrophysical Journal*, **19**, 190–212, April 1904.

[2] *Phil. Mag.*, (6) **7**, 456–467, May 1904.

[3] Inaug. Diss., Halle, 1905.

[4] "Line Structure, I," *Astrophysical Journal*, **23**, 64–79, January 1906.

component remains, while lines of the last type require only some simple special assumption as to structure for explanation.

The commonest types of line growth are shown in the accompanying figures. Lines of type (2), remaining single until they reverse, possess a luminosity curve either like an inverted ∧ (Fig. 1), or like an inverted ∩ (Fig. 2). The former has diffuse wings, a bright center, and disappears as a very narrow *line*. The latter is broad

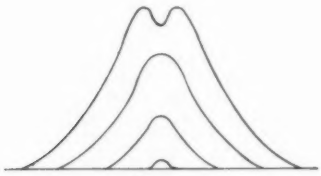


FIG. 1

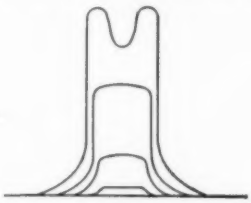


FIG. 2

and uniform in brightness, has barely perceptible wings, and disappears as a *ribbon* of light. *Cu* λ 5218 is characteristic of the former, *Cu* λ 5104 of the latter type. The iron lines reverse somewhat as shown in Fig. 3. They twin at an early stage, appearing exactly like a pair of adjacent lines with very sharp edges. As the intensity is increased, the components retreat from each other, broaden, and finally show reversal proper. The figures exaggerate the actual effects and ignore second-order phenomena like superposed structures; but only the effects shown are to be discussed in this paper.

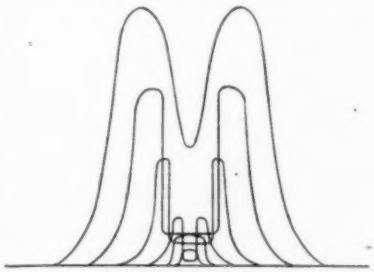


FIG. 3

The resolving power of an echelon grating is sufficient to justify considerable detail in the description of the structure of a line 0.2 t.–m. broad. An echelon of thirty plates, each 10 mm thick and differing in width by ½ mm from its neighbors, has a dispersion approximately the same as that of a 21-foot grating in the second order. Its resolving power is about ten times as great as that of such a grating, being equal to the distance between adjacent orders divided by the number of plates. Calculation indicates that the narrowest lines would appear 0.010 t.–m. broad, but one of the satellites of

the green mercury line appears considerably narrower than this limit.

As to its truthfulness, the echelon appears to be comparable with the prism and far superior to an ordinary reflection grating, giving always a normal spectrum free from false lines. Independent observers using instruments from different makers obtain results in the highest possible agreement.

LIMITING CASES OF REVERSAL

Before discussing line structure as dependent upon the individual sources, let us examine carefully the effects that might be produced by lack of homogeneity in the radiating body of gas. Might not such doubling as is shown in Fig. 3 be merely an extreme case of reversal by absorption, not involving twin modes of vibration in each source nor the existence of two different classes of radiators? Assuming Kirchhoff's law

$$E=AJ\,,$$

and the law of non-homogeneity

$$E_{12}=E_1+E_2-A_1E_2\,, \qquad (1)$$

and extreme cases of radiation (E) and absorption (A) from the layers of gas 1 and 2, what would be the extreme types of line structure possible?

The condition for reversal usually given[1] is that the emission E_{12} from two layers of gas shall be less than the emission E_2 from the rear layer alone. This condition gives

$$E_1<A_1E_2 \text{ or } E_2>J_1\,; \qquad (2)$$

hence for reversal the emission of the rear layer must be greater than the front layer could emit if it were infinitely thick, or greater than a perfect radiator or black body could emit if in thermal equilibrium with it.

But this condition, while necessary, is by no means sufficient; with this condition holding, the effect of the absorbing front layer might be merely to pare off the tip of the emission curve E_2 without any reversal at all. To be sufficient a more general condition must be imposed.

[1] See Kayser, *Handbuch der Spectroscopie*, **2**, p. 53.

Reversal may be observed when there is a minimum between two maxima in the emission wave-length curve. Hence the necessary and sufficient condition for reversal is that the wave-length derivative of equation (1) shall change sign more than once, that is, shall have not one but three real roots in the neighborhood of the maximum. The derivative of (1) is

$$\frac{dE_{12}}{d\lambda}=\frac{dE_1}{d\lambda}\left(1-\frac{E_2}{J_1}\right)+\frac{dE_2}{d\lambda}\left(1-\frac{E_1}{J_1}\right)+\frac{dJ_1}{d\lambda}\frac{E_1E_2}{J_1}\,, \tag{3}$$

which for convenience in discussion may be written

$$E'_{12}=E'_1L+E'_2M+J'_1N\,. \tag{4}$$

The roots of the right-hand member of (3) give the necessary and sufficient conditions for all kinds of reversal from a heterogeneous radiator of two layers. Complete reversal, auto-reversal, reversal due to Doppler shift, and many other kinds are special cases dependent upon various differences in nature, state, and thickness of the two layers.

Now the derivatives E'_1, E'_2, and J'_1, may be either positive, negative, or zero, and that, except in special cases, independently of one another. By the necessary condition (2), E_2 must be greater than J_1, hence L is always negative. M is always positive, since E_1 can never be greater than J_1. N is essentially positive.

When the rear layer is an incandescent solid, conditions give $E_2=J_2>J_1$ at or near maxima of E_1; hence E'_{12} is opposite in sign from E'_1 for all wave-lengths near maxima. Reversal is most pronounced when E_1 approaches J_1 in value, since this condition—absorbing layer thick—makes E'_{12} a maximum. This is the case of complete reversal. Auto-reversal occurs when both layers are of the same kind of gas, and hence E_1 and E_2 have identical roots. In this case reversal occurs at or near (depending upon the magnitude of the final term) these roots when $E_2>J_1$, whether E_1 be small in comparison with J_1 or nearly equal to it. Reversal will be symmetrical only when the greatest positive and negative values of E'_{12} are equal in magnitude, and this is a very special case. Equation (4) shows that dissymmetry may be caused by differences in thickness or of state of the emitting layers, or even by the mere position of the maxima relative to the maximum of the correspond-

ing black-body curve J_1. If the two layers are identical in all respects, E_{12} can have no maximum or minimum other than those of E_1 and E_2; hence lines from a homogeneous source cannot be reversed. In the capillary of a Plücker tube, viewed from the side, conditions are such that E_1 and E_2 have the same roots, but E_1 is vanishingly small. Hence by (4) reversal requires that not only must E_2 exceed J_1 but $E_1'' L$ must exceed E_2''. That is, the curvature of the emission curve E_2 must be less than L times the curvature of E_1. Since, under the conditions assumed, E_2 can be little if any greater than J_1, L is small, and reversal cannot be deep and can occur only under extreme conditions.

Consider the conditions necessary for the deep, sharp reversal illustrated in Fig. 3. The sharp definition of the components requires large values—approaching infinity—of the derivative E_{12}', while depth of reversal requires that E_{12} have very small values—approaching zero—at the minimum. But by (4) E_{12}' cannot be greater in absolute value than either E_1' or E_2' alone, since L and M are of opposite sign and J_1' is small. Hence a reversal cannot be more sharply defined than an unreversed line of equal intensity. Further, by equation (1), E_{12} cannot be less than E_1 at any wave-length, but deep reversal requires that E_1 be large. Both this and the preceding condition are of doubtful existence so far as observations go, so that the evidence is all in favor of the early doubling of the iron lines being due to causes other than absorption.

A complete discussion of reversal will be possible only after the function E has been constructed. The emission E is a function of four classes of arguments: (1) arguments specifying the nature of the radiating body; (2) arguments like temperature, pressure, internal energy, etc., specifying its condition; (3) mechanical arguments like thickness, specifying form and dimensions; and lastly, (4) wave-length or frequency of the emitted radiation. Kirchhoff's law and the law of heterogeneity (1) hold of course whatever the forms of the general function E. While we have not sufficient data at present to construct the complete emission function of any of these arguments, it will be useful here to set up a thickness function with the understanding that it is but tentative until more exact data shall substantiate it or cause its rejection.

The partial derivative of the complete emission function must have a single real root at infinity, since it is asymptotic to $J(\lambda) =$ constant. Hence write

$$dE = (E_\infty - E)\,\phi\, d\,\theta\,,$$

where θ is the thickness of the radiating layer, and ϕ is a function of wave-length, nature of the radiator, condition of the radiator, or anything else except thickness. Integrating,

$$E = J\,(1 - e^{-\phi\theta})\,, \qquad (5)$$

provided the emission E_∞ of an infinitely thick layer is identical with the emission J of a perfect radiator or black body in thermal equilibrium with it.

As a severe test of this thickness function (5), substitute in equation (1) for two layers of thickness θ_1 and θ_2 of the same nature and in the same condition; hence $\phi_1 = \phi_2$ and $J_1 = J_2$. This substitution gives on reduction and collection,

$$E_{12} = J\,(1 - e^{-\phi\,(\theta_1 + \theta_2)})$$

as required.

In applying the thickness function (5) to the conditions for reversal expressed in (4), note that

$$\frac{dE}{d\lambda} = \frac{dE}{d\phi}\frac{d\phi}{d\lambda} = \theta J e^{-\phi\theta}\frac{d\phi}{d\lambda} \equiv \theta J \Theta \phi^1\,.$$

Hence the condition for reversal (4), expressed in terms of thickness, may be written

$$E_{12} = \theta_1 \Theta_1 \phi_1^1 (J_1 - E_2) + \theta_2 \Theta_2 \phi_2^1 J_2 \theta_1 + J_1 J_2 (1 - \Theta_1)(1 - \Theta_2)\,. \qquad (6)$$

As before, $E_2 > J_1$ is a necessary condition for reversal, hence $J_1 - E_2$ is always negative. Only the derivatives ever change sign. All other quantities are essentially positive.

Equation (6) shows the effect of the thickness of the emitting layers on the position of the reversal, its symmetry, sharpness, and depth. We may note here that increasing the thickness θ_1 of the reversing layer (a) displaces the reversal away from the corresponding black body maximum; (*b*) sharpens the reversal on the side toward the larger waves, producing the opposite effect on the other side, and hence, (*c*) producing greater dissymmetry, while (1) and (5) show (*d*) that the depth of reversal is thereby increased. The thicker both layers, the greater the values that E_{12}^1 may have

and the less the value of E_{12} at the minimum; hence the deeper and sharper the reversal.

In the capillary of a Plücker tube viewed laterally, any reversing layer must be extremely thin, if existent; hence reversal could hardly amount to more than a mere dimple in the emission maximum. Such doubling as is depicted in Fig. 3 would require a thick reversing layer to produce such depth of reversal. But a thick reversing layer would produce diffuse boundaries to the reversal, while, as a matter of fact, the edges are extremely sharp, so that here again the doubling can hardly be regarded as a reversal. If then mere heterogeneity of condition is insufficient to produce the effects observed, we must turn to the radiators themselves for an explanation.

THE EMISSION OF BROAD, DOUBLE AND MULTIPLE LINES

Our problem then is to relate not only the amount but the quality of the emission from a body to the internal energy and nature of the body. Since the line structure varies directly with the energy of the source, the radiator itself must be modified by the energy which it transforms. The broadening to be accounted for is of the order of 10^{-4} to 10^{-6}; hence is to be treated as a second-order effect.

Broadening may be due to: (1) the effect of damping by radiation on the period of the radiator; (2) disturbances caused by impact; (3) an aggregate effect of Doppler shifts due to motion of the radiator along the radius of vision; (4) Doppler effects due to rotation of the radiator; and lastly, (5) and (6), the effect of motions of translation and rotation on the elasticity of the radiator, if this is electromagnetic in its nature.

Before discussing these effects, let us consider briefly the general relations existing between frequency (p) and the inertia (m), viscosity (s), and rigidity (r) of any body subject to elastic and dissipative forces. If x, $\dot{x}$, and $\ddot{x}$ represent respectively the displacement, velocity, and acceleration of any part, then its motion will be governed by the function,

$$m\ddot{x}+s\dot{x}+rx,$$

and its frequency will be given by the roots of the equation,

$$-mp^2+isp+r=0. \qquad (7)$$

1. Broadening by damping. In the equation just written the damping factor is $s/2m$, while damping affects the period by an amount $s^2/4m$. For a green line $p=7\times10^{15}$, and if but 0.005 t.-m. broad—an extremely narrow line—$d\lambda:\lambda=10^{-5}$. Hence we have

$$\frac{dp}{p}=\frac{1}{p}\cdot\frac{s^2}{4m}=\frac{d\lambda}{\lambda}=10^{-5},$$

so that the damping $s/2m>1.4\times10^{11}$—a value at least a million times greater than the greatest which would permit of periodic motion, while interference experiments lead us to infer a damping so slight that at least 10^5 effective oscillations are executed after each excitation.

2. Broadening by disturbances caused by impact would be of the nature of faint, diffuse wings to relatively sharp and narrow lines, since all kinetic theory and spectroscopic evidence indicate that the time occupied by impact is very much less than the time elapsing between impacts, or even the time occupied in radiating except at pressures of many atmospheres. There is little, if any, evidence for broadening of this nature at atmospheric pressure or less.

3. Broadening by the Doppler effect has received considerable attention by F. Lippich[1], Rayleigh[2], A. A. Michelson[3], and others. If V is the velocity of light and u is the component of the velocity of the radiation along the visual radius, then by Doppler's principle

$$\frac{d\lambda}{\lambda}=\frac{u}{V}=10^{-5}$$

for a green line 0.05 t.-m. broad. This gives $u=10^5$ cm/sec, a velocity which kinetic theory assigns to the molecules of gases at ordinary temperatures and pressures. The aggregate of such effects would be an emission curve E resembling the probability curve in form, and having a mean width varying roughly as the square root of the internal energy and inversely as the square root of the density. The molecular Doppler effect then appears sufficient to account for simple broadening, but could not cause twinning or any structure other than that of the simple probability-curve type.

[1] *Pogg. Ann.*, **139**, 465–479, 1870. [2] *Phil. Mag.*, (5) **37**, 296–304, 1889.
[3] *Astrophysical Journal*, **2**, 251–263, 1895.

4. Rotatory motions of the radiator might give a Doppler effect similar to the effect of translation, but only a tenth to a hundredth as great, hence would be masked by the latter and need not be considered here.

5. If the generator of radiation is a distorted aggregate of electrical charges or charged particles, we may calculate the order of the effect of their motion of translation on their rigidity and hence on their frequency. Electromagnetic theory shows that the electromagnetic forces tending to preserve the configuration of such an aggregate always contain a factor varying from

$$1-\frac{u^2}{V^2} \text{ to } \left(1-\frac{u^2}{V^2}\right)^{\frac{1}{2}},$$

according as the force is acting in a line parallel or transverse to the line of motion of the aggregate as a whole. This factor would affect the frequency by an increment varying from a half to a fourth of u^2/V^2, since the frequency varies as the square root of the rigidity r and u^2/V^2 is small compared with unity. Hence for a moderate broadening, $d\lambda:\lambda=10^5$, we have

$$\frac{dp}{p}=\frac{u^2}{V^2}=\frac{d\lambda}{\lambda}=10^{-5}$$

and hence for

$$V=3\times10^{10},\ u=10^8 \text{ cm/sec.}$$

Since an equal broadening by the Doppler effect would require a mean velocity of but 10^5 cm/sec., it appears that the electromagnetic effect would be completely masked by the Doppler effect.

6. The effect of rotation on the elasticity of a radiator cannot be determined by purely mathematical means without assuming some form of structure of the radiating aggregate. This has been accomplished by J. J. Thomson[1] for the special case of a limited number of charged particles arranged in a ring and rotating about its axis. Rayleigh[2] has considered the case of an atom containing a very large number of negatively charged particles. Both writers have developed frequency formulas depicting spectral series, but without considering second-order effects involving the width of lines.

[1] "The Structure of an Atom," *Phil. Mag.*, (6) **7**, 237–266, March 1904.

[2] *Phil. Mag.*, (6) **11**, 117–123, January 1906.

Without assuming any particular structure, consider the effects that rotation may have on the frequency of a radiator. A neutral aggregate cannot radiate, whatever its motion, unless it possess a finite external field. It appears to be both necessary and sufficient that gases at ordinary temperatures and pressures be constituted of such neutral aggregates. A gas composed of aggregates possessing a finite external field on account of either structure or lack of neutrality, could possess permanent thermal equilibrium only when subject to very moderate acceleration. Neutral aggregates may acquire a temporary external field by distortion or a quasi-permanent field by the addition or removal of a charge or charged particle, quasi-permanent referring to a time long in comparison to the time required for recovering from distortion.

Once possessing an external field, an aggregate may in general give rise to three different types of radiation corresponding to the three possible types of acceleration. (1) Oscillations of configuration about a mean configuration, corresponding to the different modes of vibration of the aggregate. Relations between frequencies would depend entirely upon the structure of the aggregate. Radial and tangential distortion would, in general, give rise to two distinct classes of related frequencies. (2) Rotation of an aggregate about any axis would give rise to radiation having a frequency equal to the (temporary) angular velocity of the radiator. If this angular velocity varies from time to time, or if neighboring aggregates differ in angular velocity, then a mass of aggregates will give a radiation frequency curve resembling the probability curve, having corresponding parameters. (3) Rectilinear acceleration, if sufficiently great, would give rise to aperiodic radiation. The radiation of this type from a mass of aggregates would be distributed about a maximum according to the probability law.

In any case the acceleration—central or linear—must be of the order of 10^{20} cm/sec.^{2} or more before the amount of the radiation becomes an effect of the first order. The intensity of the radiation from each generator is roughly proportional to both velocity and acceleration.

Consider now the effect of rotation on the frequencies of the radiation due to distortion. Neglecting effects of lower order, a

slight increase or decrease δr in the rigidity r of an aggregate will in general cause a slight increment or decrement δp to the frequency of each mode. Hence if p_0 be a frequency when stationary, the frequency when in rotation will be p_0 plus or minus an increment $\omega = \delta p/2$, or

$$p = p_0 \pm \omega. \tag{8}$$

For a pure oscillation of configuration without rotation $p = p_0$, since in this case the rotation, and hence the effect of rotation, is null. On the other hand, for a pure rotation without distortion, p_0 disappears and $p = \omega$; hence ω is to be identified with the angular velocity-frequency of the radiating aggregate. The identity of ω is to be regarded as mathmetically only tentatively established, but physically it appears to be both necessary and sufficient.

For a line half a tenth-meter wide in the green $d\lambda . \lambda = 10^{-4}$; hence for ω we have

$$\frac{dp}{p} = \frac{\omega}{p} = \frac{d\lambda}{\lambda} = 10^{-4},$$

from which $\omega = 3 \times 10^{11}$. For an aggregate of 10^{-8} cm radius this angular velocity corresponds to a rim speed of 3×10^3 cm/sec.—of just the order to be expected in a gas in which the mean relative velocity is 10^5 cm/sec. and rotation is caused by grazing impact. Thus the observed broadening is not greater, is in fact somewhat less, than is to be expected from equation (8) with ω identified as the angular velocity of the atom.

A frequency $\omega = 3 \times 10^{11}$ corresponds to a wave-length $\lambda = 0.6$ mm $= 600\mu$. But the broadening of a spectrum line by (8) does not necessarily imply a coexistent emission band far in the infra-red, for a frequency of 3×10^{11} is too slow to communicate an appreciable amount of energy to the surrounding ether. This will first become appreciable when the angular velocity increases to the order of about 10^{13}.

Orientation can evidently have no effect on the change of frequency caused by rotation. Hence the radiation from a large number of aggregates will be a mere summation of the radiation from individuals, unmodified by orientation. Since each rotating radiator generates waves of two nearly equal frequencies (with or without the original mean frequency according to structure),

the radiation from a large number of radiators will consist of double or triple lines, each broadened by the variation in angular velocity of the radiators, and further broadened by the Doppler effect of translation when this is relatively great. As the mean velocity of rotation is increased, the components of the doublet or triplet will broaden and move farther and farther apart according to the law expressed by (8).

If then, the structure of a molecule be such that its angular velocity or the angular velocity of its component atoms be large in proportion to its linear velocity, the spectrum lines emitted will double or triple when faint, and continue so with broadening and separating components as the excitation is increased, until reversal occurs on account of the non-homogeneity of the source (Figs. 3 and 4). The lines of the iron spectrum are typical of this behavior. If, on the other hand, the angular velocity is small compared with the linear velocity, the Doppler broadening will mask the doubling caused by rotation, and lines will remain single until reversal proper occurs (Figs. 1, 2, and 5). In this case the effect of rotation will be to broaden the upper part of the emission wave-length curve,

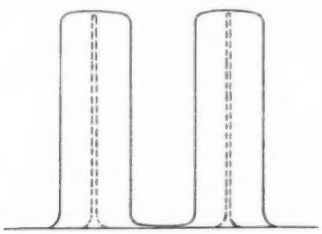


FIG. 4

Dotted line = effect of pure rotation.
Solid line = effect of rotation and translation.

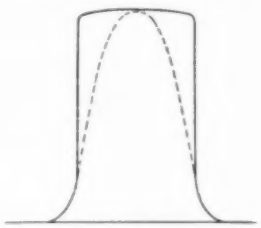


FIG. 5

Dotted line = effect of pure translation.
Solid line = effect of translation and rotation.

to change it from the inverted $\wedge$ to the inverted $\cap$ type. Fig. 1 shows the type of line structure corresponding to an angular velocity negligibly small in comparison with the linear velocity. Fig. 3 shows the opposite extreme corresponding to large angular and small linear velocity. Comparing the broadening with the doubling, we see that doubling will be apparent when the expression $\omega V : up$ $(= 2\pi\lambda\omega : u)$ is large compared with unity, while a line will broaden without doubling when this is small compared with unity.

Since they develop as depicted in Fig. 1, it would appear that the lines of the sodium group come from atoms or molecules having a relatively small angular velocity, while for the iron and platinum group the angular velocity is large. If these deductions are correct, spectroscopy may supply valuable data for the kinetic theory of gases and the theory of magnetism.

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AN ATTEMPT TO FIND THE CAUSE OF THE WIDTH AND OF THE PRESSURE-SHIFT OF SPECTRUM LINES

By W. J. HUMPHREYS

The widths of all spectrum lines and the wave-lengths certainly of many are both functions of the pressure surrounding the luminous source, and the following is an attempt to develop a working hypothesis for the explanation of these phenomena.

Many spectrum lines yield the well-known Zeeman effect when their source is in a magnetic field, and therefore we conclude that the particles producing these lines have, at least while luminous, magnetic fields of their own. The nature and the magnitude of this phenomenon indicate that the radiations and the magnetic fields are due to moving electrons of the cathode type, and it is convenient, in discussing the Zeeman effect, to regard these electrons as revolving in closed orbits.

While such an assumption seemed desirable, it was necessary to give it proper limitations, to find a structure meeting these conditions that would not violate known physical laws; and this problem has been solved by J. J. Thomson,[1] who has shown that an atom consisting of negative electrons in a sphere of uniformly distributed positive electricity is stable when the electrons are properly spaced and rapidly rotating in coaxial circular orbits, the axis of rotation passing through the center of the atom, and the number of electrons per orbit increasing with the order of its distance from the center. When undisturbed the angular distance, as seen from a point on the axis, from electron to electron of any given ring is a constant.

Clearly, if the electrons are revolving in circular orbits, each ring will produce its own magnetic field, and therefore the system will possess minimum potential energy when the planes of the rings are parallel and all the electrons are rotating in the same direction. I therefore assume that this is the general distribution and condi-

[1] *Philosophical Magazine*, (6) **7**, 237–265, March 1904.

tion of the electrons in an atom. As to the equal amount of positive electricity, it is only necessary that it shall be approximately at rest with reference to the electrons, and so distributed as to secure the stability of the rings. This may be secured, as above explained, by having it uniformly distributed throughout a sphere whose center is on the axis of the rings, and whose radius is equal to or greater than that of the outermost orbit. Possibly the same result would follow if the positive electricity was uniformly distributed throughout an ellipsoid of revolution whose equatorial plane was parallel to the planes of the orbits. But it is sufficient to know that the ring system of negative electrons is possible.

I assume that the frequency of the electromagnetic or light waves given off by any ring of electrons is either the same as the frequency of its orbital revolution, or else directly dependent thereon. For calculations the former will be used. I assume further that the frequency of this orbital revolution is not appreciably, if at all, a function of the kinetic energy of the atom as a whole—that it is practically free from temperature changes. This may seem to violate certain teachings in the kinetic theory of gases, but practically the same contention has been made by J. J. Thomson,[1] who says:

> If the kinetic energy arising from the motion of the corpuscles relatively to the center of gravity of the atom could by collisions be transformed into kinetic energy due to the motion of the atom as a whole, i. e., into molecular temperature, it would follow from the kinetic theory of gases, since the number of corpuscles in the atom is exceedingly large, that the specific heat of a gas at constant pressure would be very nearly equal to the specific heat at constant volume; whereas, as a matter of fact, in no gas is there any approach to equality in these specific heats. We conclude, therefore, that it is not by collisions that the kinetic energy of the corpuscles is diminished.

In connection with this question see also Magie, "The Partition of Energy," *Science*, N. S., **23**, 168 *et seq.* Besides, if it is granted that spectrum lines depend for their origin upon orbital rotations of electrons, and the frequency of these in turn upon the temperature of the gas, then their wave-lengths and the magnitude of the Zeeman effect should be corresponding functions of the temperature; but such a relation does not appear to exist.

[1] *Electricity and Matter*, pp. 98, 101, 104, and 105.

Again, probably the existence of magnetism in a piece of metal owes its origin to Ampere currents in the form of permanently rotating rings of electrons. If so, then no temperature, however low, has stopped this rotation.

Consider now some of the mutual actions of this type of atom upon each other when in a gaseous condition; to be specific, say iron atoms in the electric arc, at atmospheric pressure.

The best available sources give for iron atoms, under the specified conditions, the following data:

Average velocity of each atom as a whole, 10^5 cm per second.
Charge of each electron, 10^{-20}, in electromagnetic measure.
Charge of each electron, $3 \cdot 10^{-10}$, in electrostatic measure.
Total number of electrons per iron atom $5 \cdot 10^4$.
Mass of each electron 10^{-27} gram.
Mass of iron atom $5 \cdot 10^{-23}$ gram.
Radius of atom, equal to or greater than radius of outer ring of electrons, 10^{-8} cm.
Average distance between centers of atoms, $6 \cdot 10^{-7}$ cm.
Assumed number of orbital revolutions per second, inversely proportional to the wave-length; 10^{15} for λ 3000, $5 \cdot 10^{14}$ for λ 6000.

This angular velocity does not seem to be at all prohibitively great. The "centrifugal force" on a single electron is equal to $m\omega^2 r$, where m is its mass, ω its angular velocity, and r the radius of its orbit; while the force with which it is drawn by the positive charge toward the center of the atom is equal to $\frac{e^2 5 \cdot 10^4 r}{a^3}$, where e is the electrostatic charge on each electron, and a the radius of the atom. The ratio of these two forces is independent of the radius of the electron orbit, and, with the attraction for the numerator, is equal to $2 \cdot 10^7$. Consequently the electron rings might have angular velocities of the order assumed, and still be in no danger of flying to pieces.

From the above data it is easy to calculate approximately the magnetic force at the center of the atoms, but to further simplify the work it will be assumed that the several rings of electrons are coplanar and concentric with the atom.

Let i_k be the average current, in absolute electromagnetic meas-

ure in the ring K, then $i_k=\frac{Q_k}{t}$, where Q_k is the total quantity of electricity passing any point on the orbit in the time t.

Let ω_k be the constant angular velocity, then $i_k=n_k 10^{-20}\frac{\omega_k}{2\pi}$, where n_k is the number of electrons in the ring K. Let the radius of this ring be r_k, then the magnetic force at its center is

$$H_k=\frac{n_k 10^{-20}\omega_k}{r_k}.$$

Therefore the entire system of orbits gives at the center of the atom a magnetic force

$$\Sigma H_k=\Sigma\frac{n_k 10^{-20}\omega_k}{r_k}.$$

But for λ 6000, $\omega=\pi 10^{15}$, probably somewhat less than the average, though of the same order, while the radius of the outer ring is not greater than 10^{-8} cm, and the radii of the others still less. It therefore would seem conservative to let the average angular velocity be $\pi 10^{15}$, and the average radius 10^{-8} cm. In this case the magnetic force at the center of iron atoms is

$$H=\frac{5\cdot 10^4 10^{-20}\pi 10^{15}}{10^{-8}}=5\pi 10^7.$$

According to the above conception of the structure and behavior of atoms, the greater the atomic weight, the greater the strength of the magnetic field, though not necessarily in exactly the same ratio.

It may also be worth while to note that the gyroscopic action of the whirling electrons will tend to cause an atom, when it moves about, to keep its axis always pointing in the same direction, and that the greater the atomic weight, the greater will this tendency be. However, in the turmoil of an electric arc, the direction of an atom's axis is likely to change both rapidly and frequently.

The magnetic fields of such atoms as are here considered are still powerful at appreciable distances from the atoms themselves. Let r be the radius of the orbit, d its distance from a point p on its axis; then the magnetic force at this point, along the axis, is, for iron atoms,

$$H=5\pi 10^7\frac{r^3}{d^3}.$$

If the distance is ten times the radius, the force is $5\pi 10^4$, or threefold the most intense field yet obtained by means of an electromagnet. Even at the average distance of the nearest atom, that is, $6 \cdot 10^{-7}$ cm, $H = 730$. Of course, the direction and the magnitude of this force is determinate for any point on or off the axis, but for the purpose of this paper the simple case of points on the axis is sufficient.

With velocities of 10^5 cm per second each atom will very often approach and leave other atoms. The magnitude of their relative velocities and the angle between the planes of the electron orbits both will vary through wide ranges, but in general at every approach and recession each atom will produce an induced current in the other of greater or less magnitude.

Consider the case of two atoms on a common axis. When their electrons rotate in the same direction, an approach will induce in each atom an electromotive force that will oppose the existing current, and as the atoms recede the induced electromotive force in each will be with the current. If, however, the electrons are rotating in opposite directions, the results will be just the reverse; that is, the induced electromotive force will be with the currents as the atoms approach, and against the currents as they leave each other. In the first case, where the electrons rotate in the same direction, the currents will decrease as the atoms approach, and then increase as they recede, coming finally to their original undisturbed values. In the second case they first increase, then decrease till again undisturbed values are reached. In the first case the currents are always less, in the second always greater, than they are in the undisturbed atom.

In the case of an induced current,

$$E = L\frac{di}{dt} + Ri \ ,$$

where E is the induced electromotive force, L the self-induction of the circuit, $\frac{di}{dt}$ the rate of change of the current, R the ohmic resistance of the circuit, and i the strength of the current.

But when, as in the present case, the circuit consists of a single turn, $E = \frac{dN}{dt}$ is the rate of change of the lines of magnetic force

inside the circuit. Therefore for the ring K, if its self-induction remains constant, the equation becomes,

$$\frac{dN_k}{dt}=L_k\frac{di_k}{dt}+R_k i_k \ .$$

But $R_k=0$, since the electrons meet with no ohmic resistance in their orbits, and as the dt on one side of the equation is identical with the dt on the other, therefore

$$di_k=\frac{dN_k}{L_k} \ .$$

Thus the induced current is at all times directly proportional to the total change in the number of magnetic lines of force passing through the circuit. Besides, every induced current persists so long as the new number of lines of magnetic force through the circuit is not allowed to change. In the case of any particular ring K, the value of the current is

$$i_k=\frac{n_k 10^{-20}\omega_k}{2\pi} \ .$$

Hence $di_k=\frac{n_k 10^{-20}d\omega_k}{2\pi}=\frac{dN_k}{L_k}$, which shows that $\frac{d\omega_k}{dN_k}$ is a constant. Therefore, if the period of a light vibration is the same as the period of orbital rotation of the electrons, or directly dependent upon it, then, whatever the magnitude of the Zeeman effect on a given line, this effect should not be momentary, but should persist invariable so long as the field is unchanged, which, as we know, agrees with the observations.

Again, if the change of the magnetic field does not change the self-induction of the circuit—does not change the radii of the orbits—then the change in ω, the magnitude of the Zeeman effect, should be directly proportional to the strength of the magnetic field, which also is in agreement with observation.

That only the angular velocity, and not the orbital radius, varies when the magnetic field is changed has been demonstrated very cleverly by Langevin.[1] The importance of this proof, I trust, will justify the following slightly modified reproduction of it.

$$M=iA=\frac{ne\omega r\pi r^2}{2\pi r}=\frac{ne\omega r^2}{2} \ , \qquad (1)$$

[1] *Journal de Physique*, (4) **4**, 678–692, October 1905.

where M is the moment of the magnetic shell equivalent to the given ring, A its area, i the current, e the charge of a single electron, n the number of electrons in the ring, ω its angular velocity, and r its radius.

Let the external magnetic force, normal to the plane of the electron orbit, be changed at a uniform rate by an amount H. This will inductively change the value of the current by acting tangentially along the ring on each electron with a force Fe, so that the moment of the entire force on the ring becomes $Fner$. But if m is the mass of each electron, then the moment of the total tangential force may be written

$$2mn\frac{d}{dt}\left(\frac{\omega r^2}{2}\right).$$

Therefore

$$Fner = 2mn\frac{d}{dt}\left(\frac{\omega r^2}{2}\right).$$

From (1) $\dfrac{dM}{dt} = ne\dfrac{d}{dt}\left(\dfrac{\omega r^2}{2}\right) = \dfrac{Fne^2r}{2m}$. Hence $dM = ned\left(\dfrac{\omega r^2}{2}\right)$.

But ds on the orbit $= \omega r dt$, and therefore

$$dM = \frac{Fne^2ds}{2m\omega}.$$

Let δM be the minute change in moment during one orbital revolution. ω will remain practically constant during this short time, and therefore

$$\delta M = -\frac{ne^2}{2m\omega}\int_0^{2\pi r} Fds, \quad \text{very nearly.}$$

But the integration of an electric force around any closed circuit is equal to the rate of change of the lines of magnetic force in the circuit. Therefore $-\displaystyle\int_0^{2\pi r} Fds = \frac{dHA}{dt} = \frac{\delta HA}{\tau}$, where δHA is the change in magnetic flux through the circuit during the periodic time τ. Consequently

$$\Delta M = -\frac{ne^2HA}{2m\omega\tau} = -\frac{ne^2HA}{4\pi m}.$$

In the case of a mass m rotating uniformly in a circular orbit

under a central force $f(r)$, we have, when it is free from external disturbance,

$$f(r)=m\omega^2 r .$$

Now let a magnetic field of intensity H be established normal to the plane of rotation. This will give an added central force $He\omega r$ to each electron. As a result, suppose both r and ω to be slightly changed, giving

$$f(r+\Delta r)=m(\omega+\Delta\omega)^2(r+\Delta r)+He\omega r .$$

Therefore

$$[f'(r)-m\omega^2]\Delta r=2m\omega r\Delta\omega+He\omega r , \quad \text{nearly.} \tag{2}$$

As shown above, $dM=ned\left(\frac{\omega r^2}{2}\right)$, or for a single electron

$$\Delta M=e\Delta\left(\frac{\omega r^2}{2}\right)=-\frac{e^2HA}{4\pi m} .$$

But

$$A=\pi r^2 ,$$

hence

$$\frac{\Delta\omega r^2}{2}=-\frac{eHr^2}{4m} .$$

Also

$$\frac{\Delta\omega r^2}{2}=r\omega\Delta r+\frac{r^2\Delta\omega}{2} .$$

Therefore

$$-r\omega\Delta r=\frac{r^2\Delta\omega}{2}+\frac{eHr^2}{4m} ,$$

and

$$-4m\omega^2\Delta r=2m\omega r\Delta\omega+He\omega r . \tag{3}$$

Subtracting (3) from (2), we get

$$[f'(r)+3m\omega^2]\Delta r=0 ;$$

hence either $\Delta r=0$, or $f'(r)+3m\omega^2=0$.

If $\Delta r=0$, then from (3) $\Delta\omega=-\frac{He}{2m}$.

If, however, Δr is not zero, then $f'(r)+3m\omega^2=0$, which, combined with $f(r)$, gives $\frac{f'(r)}{f(r)}=-\frac{3}{r}$, and $f(r)=\frac{C}{r^3}$. That is, the only central force that will allow r to change is the improbable one that varies inversely as the cube of the distance. From this it would appear that r almost certainly does not appreciably change as the strength of the magnetic field is varied. If r is constant,

it follows that the self-induction is also constant, and the expressions obtained above for the induced currents are justified.

In discussing the motions of atoms among each other, it is necessary to take into consideration not only the effects of temperature, but also the results of their mutual magnetic and electrical attractions and repulsions. But each atom may be regarded as a negative ring with a positive center (center of the positive sphere) of equal amount, and therefore, at distances large with reference to the atomic radius, the resultant electrical force of one atom upon another is practically zero; but with this distance sufficiently decreased, the resultant force (repulsion) is relatively very large. That is, the electrical field of a neutral atom is practically zero everywhere except in the immediate neighborhood of the atom itself.

If u is the velocity of an electron in its ring, then the strength of the magnetic field which it produces at the point r, θ, is $\frac{eu}{r^2}\sin\theta$, and the magnetic attraction between two such moving electrons, provided their velocity does not approach that of light, is $\left(\frac{eu}{r}\right)^2 \sin\theta\cos\alpha$, where α is the angle between their directions. But the electrostatic repulsion, under the same conditions, between these electrons is $\left(\frac{e}{r}\right)^2$. In the first case e is in electromagnetic measure, in the second in electrostatic, and the ratio between these is V, the velocity of light. Therefore even in the most favorable case, where $\sin\theta$ and $\cos\alpha$ are both equal to unity, the repulsion is greater than the attraction in the ratio of the square of the velocity of light to the square of the velocity of the electrons. But the velocity of the electrons is roughly $\pi 10^{-8}10^{15}$ cm per second, or one six-hundredth that of light; and consequently, if the rings existed alone, their repulsion would greatly exceed their attraction. However, owing to the structure of the atom, as explained above, the total repulsion probably does not exceed the attraction, under most favorable conditions, except when the atoms are exceedingly close together. Of course, each force produces its own effect, independent of all the others, and therefore may be considered alone.

Take the very special but simple case of two atoms whose electrons are revolving around a common axis, and let the direction

of their planes remain fixed, just as the gyroscopic action of the whirling electrons tends to keep them. In this case, whatever the magnitude of the electrostatic action and of the temperature velocity, both will be independent of the direction of the electron rotation. The magnetic force, however, will be attraction when the directions of rotation are the same, and repulsion when they are opposite. It therefore becomes desirable to calculate the approximate velocity the magnetic force can give the atoms during the short time any two can remain sufficiently close together.

As already shown, the force at a point p on the axis distant d from the electron ring is

$$H=\frac{2\pi ir^2}{d^3}=\frac{2\pi ir^2}{(r^2+x^2)^{3/2}},$$

where x is the distance of p from the center of rotation. Let the plane of the second atom pass through p, then, if the field is uniform, the magnetic flux through the second circuit due to the first is

$$\frac{2\pi^2 ir^4}{(r^2+x^2)^{3/2}},$$

and the force between the two circuits is equal to the current in the second multiplied by the differential of the magnetic flux through it with respect to the distance separating them, which gives, when the atoms attract each other,

$$F=2\pi^2 i^2 r^4 \frac{d}{dx}\frac{1}{(r^2+x^2)^{3/2}}=-\frac{6\pi^2 i^2 r^4 x}{(r^2+x^2)^{5/2}}.$$

Applying this equation to the case of two iron atoms, the distance between whose centers is $10r$, we get

$$F=\frac{6\pi^2(25\cdot 10^{-2})^2(10^{-8})^4 10^{-7}}{[(10^{-8})^2+(10^{-7})^2]^{5/2}}=375\cdot 10^{-6}\text{ dyne}.$$

But the mass of an iron atom is $5\cdot 10^{-23}$ gram. Therefore, if one atom is kept stationary and the other allowed to move under the magnetic force at this distance, its acceleration will be $375\cdot 10^{-6}$ cm per second. Now let one atom be stationary and let another pass by it with the average velocity of 10^5 cm per second. Let the planes of their orbits be parallel and separated, at their nearest approach, by ten times the radius, and suppose the magnetic force to act while a distance of only three times the diameter is passed over.

The time required to travel this distance of $6 \cdot 10^{-8}$ cm is $6 \cdot 10^{-13}$ seconds, and in this time the velocity generated, due to the magnetic action alone, is $45 \cdot 10^{5}$ cm per second, or forty-five times the average temperature velocity. Both atoms will be acted on simultaneously, and hence their relative acceleration will be doubled. It is likely that no such great velocities would be generated under the supposed conditions, because every increase in the velocity will correspondingly decrease the period during which acceleration can take place. However, it is clear that the line-of-sight motion of such atoms will, when they are close together, be greatly different from that due to temperature alone. Besides, they certainly will act inductively upon each other, and with distinctly greater effect when they attract, thus getting into the stronger portions of each other's magnetic fields, than when the reverse is the case.

The relative velocities and orientation of the atoms will vary through wide ranges, but from the conclusions reached in the above discussion it appears that atoms can not easily, if at all, be forced into actual contact, owing to the large electrostatic forces that come into play, and that therefore when they rush close together they will rebound; also that, owing to their powerful magnetic fields, they will pursue paths that bring mutually attracting faces relatively close together, and keep the repelling ones comparatively far apart.

From these considerations certain general deductions may be made in regard to spectroscopic phenomena.

I. Only neutral atoms and positive ions—that is, atoms less one or more electrons—can give spectrum lines. The forces acting on these two classes would not be the same, and the lines themselves might therefore radically differ, as Stark[1] has shown them to. Free electrons could be expected to produce only electromagnetic pulses (Roentgen rays) as a result of their more or less sudden accelerations.

II. When a luminous gas is very attenuated, so that any given atom spends much the greater portion of its existence away from the influence of other atoms, then its spectrum lines will be nar-

[1] *Physikalische Zeitschrift*, **25**, 892, 1905.

rowest; such widths as they do have will be due to motion in the line of sight.

Neglecting collisions, we get the approximate result very simply. Let λ be the wave-length when there is no motion, λ_1 its modified value produced by line-of-sight velocity. Then $\lambda - \lambda_1 \equiv \delta\lambda = \frac{\lambda v \cos\theta}{V}$, where v is the velocity of the particle (observer stationary), V the velocity of light, θ the angle between the path of the particle and the line of sight and having all values from 0 to π. But $v = k\sqrt{\frac{T}{m}}$, where T is absolute temperature, m the mass of the luminous particle, and k a constant. Therefore

$$\delta\lambda = \frac{\lambda k \sqrt{T} \cos\theta}{V\sqrt{m}} .$$

That is, the spreading will be:

1. Symmetrical about the point of maximum intensity.

2. Proportional to the wave-length.

3. Proportional to the square root of the absolute temperature directly.

4. Proportional to the square root of the atomic weight inversely.

III. When the density becomes more and more pronounced, a correspondingly increased proportion of light will be produced while the atoms are close enough markedly to affect each other. There will then be three causes of broadening:

a) The temperature line-of-sight motion.

b) The line-of-sight motion due to the attractions and repulsions of the atoms—probably in many cases *b*) exceeds *a*).

c) The mutually induced currents in the atoms.

Both *a*) and *b*) will produce symmetrical broadening, but *c*), owing to the fact that its effect is greatest when the atoms are arranged so as to attract each other (the atoms then getting much closer together and producing correspondingly greater mutual inductions), will cause unsymmetrical broadening; the bulk of the spreading due to this cause being toward the side of longer wave-length.

Among the spectroscopic phenomena, therefore, which a dense gas may be expected to give are the following:

1. All lines should increase in width when the pressure about their source is increased.

2. With increase of pressure the maxima of all lines should shift toward the red end of the spectrum. (This is a general statement, possible exceptions will be mentioned below.)

3. Ordinarily the spreading of a line should be much greater than its pressure displacement, though with a short exposure it may appear on a negative as a displaced narrow line.

4. Since the intensity of a magnetic field due to a circular current varies inversely as the cube of the distance from it, while the average distance between the molecules of a gas (supposed monatomic in the electric arc) varies directly as the cube root of the pressure, therefore the spreading and the shift both should be roughly linear functions of the pressure. That is, they should increase approximately as the average mutual disturbing influence of the atoms, or as the intensities of those parts of each other's magnetic fields in which they exist.

5. For lines of a given element, similar in their nature, the shifts should be proportional to the wave-length.

6. The spreading and the shifting both should obtain whatever the nature of the surrounding gas, as all atoms present are supposed to be magnetic. The shift, however, should increase as the atomic weight of the surrounding medium is increased—heavier atoms possessing stronger magnetic fields.

7. The shift of analogous lines due to different elements of the same general nature, elements of the same Mendelejeff group, should increase with increase of atomic weight, since the strength of an atom's magnetic field probably is an increasing function of atomic weight.

8. Any group of lines due to a given element, such as a series, that gives Zeeman effects proportional to the wave-length, should shift under pressure in the same ratio.

9. Ordinarily, lines of large Zeeman effects should show large pressure displacements, while those of small Zeeman effects should be little shifted.

10. Any lines, such as those of band spectra, that do not show the Zeeman effect, should not be displaced.

Every one of the above conclusions is in substantial agreement with experiment, both those applying to the width,[1] and those that refer to the pressure displacement.[2]

Probably all, or nearly all, the phenomena under I and II would follow from the assumption, which the Zeeman phenomenon appears to justify, that, whatever its structure, a radiating atom produces in its immediate neighborhood a powerful magnetic field—magnetic because it yields to a magnetic field, and powerful because the extent of the yielding is small, the change in wave-length being but a minute fraction of the whole. Therefore the structure assumed at the beginning of this article may not at all agree with real atoms; but such atoms would be magnetic, a property we know luminous atoms to have, and it admits of conception and discussion. Besides such an atom would behave spectroscopically very much as real atoms do behave, and therefore it may serve the double purpose of co-ordinating known phenomena, and of suggesting certain others that may be looked for, and how.

If atoms are assumed to consist in part of rotating rings of electrons, and the spectrum lines to be due to these rotations, then it becomes necessary to show why the rings radiate only under special conditions, and why their energy does not rapidly become dissipated in radiation.

It is assumed that each ring consists of a large number of electrons, symmetrically arranged when undisturbed. In this condition the loss of energy due to radiation, as J. J. Thomson has shown,[3] would be exceedingly minute.

If the negative electricity should entirely fill the rings, after the manner suggested by Lord Rayleigh,[4] then, when undisturbed, probably their radiation would be absolutely zero. In either case the loss of energy from radiation, and any restitution of it, from absorption, from flying electrons, or otherwise, might very well be beyond the power of known means for detection.

Let such atoms as these, non-radiating when undisturbed, be

[1] Michelson, *Astrophysical Journal*, **2**, 251–263, 1895.

[2] Humphreys, *ibid.*, **6**, 169–232, 1897.

[3] *Philosophical Magazine*, (6) **7**, 237, 1904.

[4] *Ibid.*, (6) **11**, 117, January 1906.

hurled violently among each other, as they are at high temperatures. Each will very frequently approach some other closely, and in so doing will cause and suffer inductive effects. In general these disturbances will not be symmetrically distributed about the axis of rotation, and thus greater or less bunchings of the electrons will be produced, and these in their rapid rotation will cause correspondingly powerful electromagnetic radiation—a decreasing radiation with zero for its limit; but, except for disturbances, of constant period, as the deformed rings recover their normal condition. In this way the real energy of radiation is traced back to the energy of the disturbing cause; to the temperature, or kinetic energy of the atoms, and probably in some cases to loose and flying electrons.

Atoms constituted and behaving as here considered would produce radiations corresponding to the period of each ring, and therefore, certainly in the case of elements of the same family, those of greater atomic weight may be expected to furnish the greater number of lines, as they do.

While the number of radiations could not be less than the number of rings, it might be much greater, because of the possibilities for combinations analogous to combination tones in sound; combinations which readily could lead to series of doublets and triplets in which the Zeeman effect would be the same from group to group of lines, but different between the separate lines of each group. And probably, if such lines actually exist, the Zeeman effect, the broadening, and the pressure-displacement together may lead to their identification. But whether this should be the result or not, such an investigation would be reasonably sure of valuable discoveries, for the spectroscopic field is a peculiarly rich one.

In closing, I wish to thank Dr. Ames, who has read this in the manuscript, for his interest and helpful criticisms.

RESEARCH OBSERVATORY, MOUNT WEATHER, BLUEMONT, VA.
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THE LUMINOSITY OF THE BRIGHTEST STARS

By GEORGE C. COMSTOCK

The prevailing opinion among astronomers admits the presence in the heavens of at least a few stars of extraordinary intrinsic brilliancy. Gill, Kapteyn, and Newcomb have under differing forms announced the probable existence of stars having a luminosity exceeding that of the Sun by ten-thousand fold or more, possibly a hundred-thousand fold, and *Canopus* is cited as an example of such a star. It is the purpose of the present article to examine the evidence upon which this doctrine is based, and the extent to which that evidence is confirmed or refuted by other considerations.

As respects *Canopus*, the case is presented by Gill, *Researches on Stellar Parallax*, substantially as follows. The measured parallaxes of this star and of *α Centauri* are respectively 0″.010 and 0″.762, and their stellar magnitudes are -0.96 and $+0.40$; i. e., *Canopus* is 3.50 times brighter than *α Centauri*. The light of the one star is therefore $3.50\left(\frac{0.762}{0.010}\right)^2$ times as great as that of the other. But since *α Centauri* has the same mass and the same spectrum as the Sun, and therefore presumably emits the same quantity of light, we may substitute the Sun in place of *α Centauri* in this comparison, and find from the expression given above that *Canopus* is more than 20,000 times as bright as the Sun. I have sought for other cases of a similar character, using a method slightly different from that of Gill. If we assume, as is commonly done, that in the celestial spaces the intensity of radiant energy varies inversely as the square of the distance, we may derive the following well-known relation that must exist between the luminosity, L, the parallax, π, and the stellar magnitude, m, of every star, viz.,

$$L\pi^2\rho^m = c \ . \qquad (1)$$

When the parallaxes are expressed in seconds of arc, the luminosities in terms of the Sun's output of light taken as unity, and when, in accordance with the best determinations, the logarithm of the light-ratio, ρ, is put equal to 0.4 and the Sun's stellar mag-

nitude is taken as -26.2, the constant c in the above expression becomes sensibly equal to $1/\sqrt{2}$. With these values I have computed for every star brighter than the second magnitude for which I have been able to find a reliable parallax determination, the resulting value of the luminosity, L, and these values will be found in Table III.

The observed parallaxes of the stars are in every case relative; i. e., they are the difference between the parallax of the star in question and those of certain fainter comparison stars. Since the π which appears in Equation (1) is an absolute parallax, we must either assume that the comparison stars are infinitely distant, or we must add to the observed relative parallax assumed values of the parallaxes of the comparison stars obtained as follows. The direction of the Sun's motion in space and the amount of that motion per annum (spectroscopically determined) being known, we may combine with these the average angular proper motion shown by stars of any given magnitude—e. g., the seventh—and obtain from the combination the average linear motion and average distance of the given group of stars. The results of investigation of this kind made by Kapteyn for stars to the ninth magnitude and extended by the present writer over an additional two magnitudes, are presented in the second column of the following short table, from which have been interpolated parallaxes of the comparison stars corresponding to the magnitudes as given by the observer.

TABLE I

Magnitude	Mean Parallax	Mean Luminosity
3.0	0".0320	87
5.0	0.0156	58
7.0	0.0093	26
9.0	0.0063	9
11.0	0.0046	3

On the average the interpolated corrections to the observed relative parallaxes cannot be far wrong, although in any particular case they may be either too great or too small. Since the comparison stars are rarely brighter than the seventh magnitude, these corrections are always less than 0".01, and since the true parallax

of the comparison star cannot be less than 0".00, we are assured that in no case will the correction make the resulting absolute parallax of the principal star too great by so much as 0".01. On the other hand, the true parallax of the comparison star may exceed the tabular value by several, or many, hundredths of a second, and probably will thus exceed it in some cases, producing a corresponding error of defect in the resulting parallax of the principal star. As a result, Table III contains no case in which the adopted parallax has been made so much as 0".01 too great in the transition from relative to absolute parallaxes. It may, and probably does, contain cases in which the adopted value has been made several hundredths too small.

To render more precise the conditions of our search for evidence that there exist stars of extraordinary brilliancy, I adopt as the definition of such stars that they shall emit 1,000 times as much light as the Sun, and, introducing into Equation (1) the value $L=1{,}000$, find the following values of the absolute parallax of such a star corresponding to the magnitudes, m, shown in the following table.

TABLE II

L	m	π
1000	2.0	0".015
1000	3.0	0.009
1000	4.0	0.006
1000	5.0	0.004

Below the second magnitude these absolute parallaxes corresponding to $L=1000$ are not greater than the mean parallaxes of the comparison stars commonly employed, and therefore if there are such very bright stars, apparently fainter than the second magnitude, their relative parallaxes are approximately zero and indistinguishable from errors of observation. Table III therefore contains all the obtainable data for our quest, and in considering these data we find at once four inadmissible cases. The negative absolute parallaxes are clearly impossible and show only the presence of sensible errors of unknown amount. We may conjecture that the luminosities of these four stars are greater than the average, but how much

greater is quite unknown, and the stars must be dropped from consideration as unavailable for the present purpose.

TABLE III

Star	Mag.	Parallax	Lum.	Star	Mag.	Parallax	Lum.
α *Eridani*	0.5	0.050	355	β *Crucis*	1.5	0.004	30
α *Persei*	1.9	0.076	43	ε *Ursae Majoris*	1.8	0.088	22400
α *Tauri*	1.2	0.117	34	*Spica*	1.4	−0.012	.....
Capella	0.2	0.088	151	η *Ursae Majoris*	1.9	−0.042	.....
Rigel	0.5	0.008	13800	β *Centauri*	1.2	0.054	160
β *Tauri*	1.8	0.067	60	*Arcturus*	0.3	0.033	996
α *Orionis*	1.2	0.031	490	α^2 *Centauri*	0.4	0.758	2
Canopus	−1.0	0.008	54950	*Antares*	1.2	0.030	525
γ *Geminorum*	1.9	−0.012	.....	*Vega*	0.4	0.090	120
Sirius	−1.3	0.376	33	*Altair*	1.1	0.240	1
Castor	2.0	0.028	288	α *Cygni*	1.6	−0.003	.....
Procyon	0.7	0.342	6	α *Gruis*	1.9	0.023	456
Pollux	1.5	0.064	87	α *Piscis Austrinus*	1.4	0.137	21
Regulus	1.8	0.032	263				
α *Ursae Majoris*	2.0	0.058	66	Mean	1.10	0".0962	3820
α *Crucis*	1.0	0.057	173				

There remain three stars to which the data assign extraordinary brilliancy as defined above, and possibly to these should be added *Arcturus*, whose luminosity comes just within the limit. To test the character of the evidence that they afford, we form the mean of the numbers in the luminosity column, and find that on the average the twenty-five stars under consideration are individually 3820 times as bright as the Sun.

Two circumstances tend to impair confidence in this value:

a) Twenty-two of the twenty-five stars have luminosities much less than the mean value, while three have values much greater than the mean. This distribution of values, improbable in itself, corresponds entirely to what has been above indicated as a probable result of the transition from relative to absolute parallaxes. A few parallaxes have been left decidedly too small, and have produced correspondingly great values of the luminosity.

b) The mean value itself, 3820, is far too great, and is made too great by the three abnormal cases. In the last column of Table I there is shown the mean luminosity of stars of the third, fifth, seventh magnitude, etc., computed from the mean parallaxes by means of Equation (1). Without necessarily attributing a high

degree of accuracy to these results, we cannot impugn their substantial value without overturning the foundations upon which rest the parallaxes and luminosities of Table III. We regard them therefore as furnishing at least a rough indication of the order of brilliancy of these stars, showing, as they ought, a progressive increase of luminosity with increasing apparent brilliancy. Through a graphical process I have extrapolated from these data a value of the luminosity for the magnitude 1.0, corresponding approximately with the mean magnitude of Table III, and find $L=135$. On account of its non-linear character, Equation (1) is not strictly applicable to the mean relation between parallax and luminosity in a group of stars, and on this account the values of L in Table III are probably somewhat too small. On the other hand, the absorption of light in its transmission through space affects the apparent luminosity of the faint stars more than that of the bright ones, and on this account the extrapolated value of L is made relatively too great. It is not feasible, at present, to evaluate the resultant total error in the concluded luminosity of the first-magnitude stars, but whatever reasonable margin of uncertainty may be attributed to the result found above, $L=135$, it remains absolutely inconsistent with the mean value, 3820, found from Table III, while it accords fairly well with the mean, 198, of all values save the three that are otherwise suspicious. The agreement is still better if *Arcturus* be dropped from the list.

If we inquire what error must be attributed to the adopted parallaxes of three suspected stars in order to remove the abnormal character from their computed luminosities and reduce these below the limit 1000, we find the following results:

β Crucis	0".015
Rigel	0.022
Canopus	0.052

The assumption that errors of this magnitude are inadmissible either in the adopted parallaxes of the comparison stars, or in the observed relative parallaxes of the three stars named, or through a combination of both sources, constitutes the entire evidence that there exists in the heavens any star whose brilliancy exceeds that

of the Sun a thousand-fold. Opposed to it stand the considerations above designated (*a*) and (*b*), and in the judgment of the author they are of preponderant weight.

Kapteyn has investigated the luminosities of the fixed stars in a paper issued as No. 11 of the Groningen *Publications*, and there embodies a part of his conclusions in the following remarkable statement: "There will be in a space which contains 2,000,000 stars of the same luminosity as that of the Sun, 1 star with 100,000 times greater luminosity than that of the Sun," 38 stars with 10,000 times greater luminosity, 1800 stars with 1,000 times greater luminosity, etc., together with more than 12,000,000 stars of smaller luminosity than the Sun. These results appear to the present writer incredible. They are consequences, and, in so far as relates to the brightest stars, extrapolations, from an arbitrarily assumed law of the distribution of stars in space—a law in support of which no evidence is adduced beyond the statement, "the best thing further to admit seems to suppose that the quantities z . . . . are distributed in accordance with the law of error." The assumption thus made leads to the conclusion that the stellar density, the number of stars per unit of volume, is a minimum in the region adjacent to the Sun, increases rapidly to a maximum at the average distance of stars of the fourth or fifth magnitude, and thereafter diminishes slowly to the supposed limits of the stellar system. Recognizing the improbability of such a variation of stellar density, Kapteyn endeavors to remove it by a suitable choice of constant coefficients in his equations, but with no considerable success. The result is inherent in his fundamental assumption, and in view of the entire lack of support for this assumption and its improbable consequences, it does not seem necessary to give to the conclusions quoted above any greater measure of credence than Kapteyn himself assigns to the assumed law from which they are derived: "I need not say that I do not take the supposition made here for an expression of the truth. I am only of opinion that by making it we may expect to get a good step beyond what may be reached by simply attributing one and the same parallax to stars of a determined magnitude and proper motion."[1]

[1] *Pub. Groningen*, No. 8, p. 21.

Upon a survey of the whole case the author is unable to find any adequate evidence that the maximum of stellar luminosity requires more than three figures for its expression. He is further of opinion that the mean luminosity of the first-magnitude stars cannot be represented by a number with less than three significant figures.

WASHBURN OBSERVATORY,
Madison, Wis.

Minor Contributions and Notes.

DETERMINATION OF RADIAL MOTIONS BY OBJECTIVE-PRISMS

In Harvard *Circular* No. 13 a method is described for determining the radial motions of stars from spectra obtained with objective-prisms. As this method has recently been criticised, it may be well to describe some recent results obtained here with the Draper telescopes. A photograph is taken in the usual way, the prism is then turned 180°, and a second exposure is given on the same plate. It is not necessary or advisable to use two plates, as recommended in *Circular* No. 13. It is often more convenient to reverse the telescope, instead of turning the prism. In the latter case the plate must also be turned 180°. The second images should be brought side by side with the first, and as near together as possible. It may be better to place the spectra end to end. They may be brought into any desired position by the aid of a ground-glass screen, or more precisely by an eyepiece with cross-hairs. Were there no errors, it would be necessary only to measure the distance apart of the corresponding lines of each pair of spectra. Each star whose radial motion was known would serve to determine the constant distance apart of the lines. The differences in distance, converted into wave-lengths, would give the required motions of the other stars. The motions of the Sun and Earth are eliminated, since they are the same for all. The principal sources for error, such as those due to the distortion of the lens and changes of temperature, are radial, and may be determined, using both co-ordinates of the lines in all the spectra. Changes in the differential refraction may be reduced, if desired, by turning the prism so that the spectra shall be horizontal, instead of vertical.

Fig. 1 was obtained from a contact print of a negative representing the *Pleiades*. It was taken on January 29, 1906, with the 11-inch Draper telescope. The scale is $52\overset{m}{.}6 = 0.1$ cm, and the exposures were 37 minutes and 30 minutes, respectively. These conditions are by no means ideal, but the dispersion is such, for the central portions of the spectra, that $1''$ corresponds to an approach or recession of the star of about 70 km in each spectrum. Since double the motion is measured, if positions

[1] *Harvard College Observatory Circular* No. 110.

can be determined with the accuracy of $\pm 0''.1$, which many astronomers claim is not a high degree of precision for determining star places, the corresponding probable error in the motion would be ± 3.5 km. About a dozen stars could be measured on this photograph, the brightest,

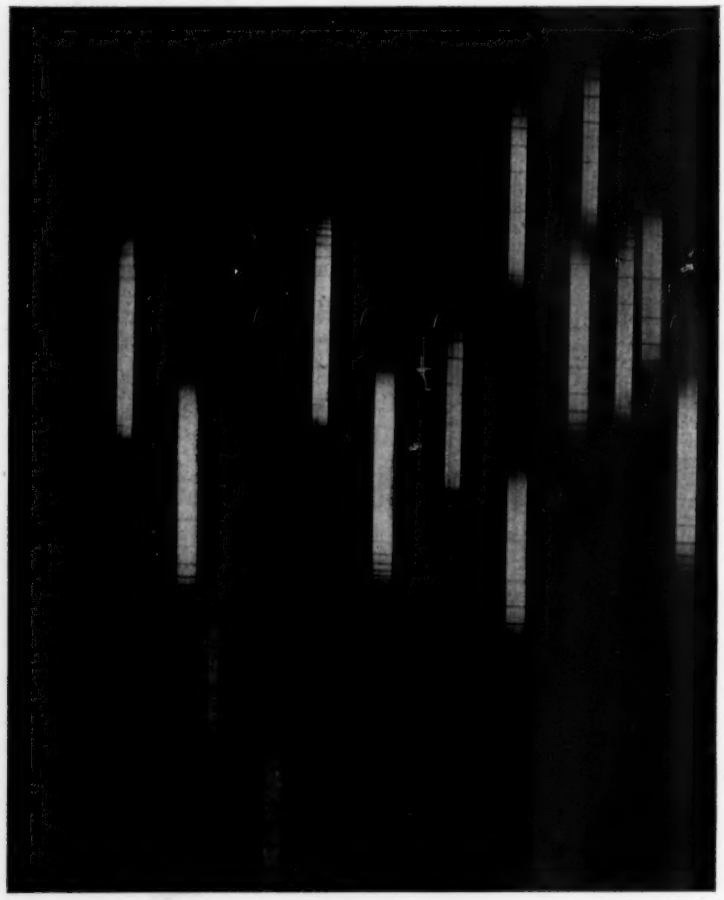


FIG. 1

η Tauri, having a magnitude of 3.0, and the faintest, 6.0. In this photograph the images are some distance apart. It is intended to illustrate the method, but not for actual measurement.

Another photograph, taken with the 8-inch Draper telescope on January 29, 1906, with exposures of 20^m, covers a region 10° square, with

the *Pleiades* as a center. The dispersion is about one-third of that in Fig. 1, but as the scale is also about one-third, the definition is much better, and measures might have nearly the same degree of precision. Much fainter stars are shown on the latter plate, lines being clearly defined in stars of the eighth magnitude. For this reason, and owing to the extent of the region covered, the number of stars measurable exceeds a hundred, but many of these could probably be used only to determine the corrections, since the distortion is large near the edges of the plate, owing to the distance from the center.

EDWARD C. PICKERING.

FEBRUARY 9, 1906.

STARS HAVING PECULIAR SPECTRA. THIRTEEN NEW VARIABLE STARS[1]

The examination of the later photographs of the Henry Draper Memorial, by Mrs. Fleming, has led to the discovery of a number of variable stars and other objects having peculiar spectra. A list of these is given in Table I, together with two additional variables found by the examination of chart plates, one by Miss E. F. Leland and one by Miss S. E. Breslin. The constellation and number in the *Durchmusterung* are given in the first two columns. The approximate right ascension and declination for 1900 and the catalogue magnitude, except in the case of variable stars, are given in the third, fourth, and fifth columns. The catalogue designations are taken from the *Bonn Durchmusterung*, except in the case of $-59^{\circ}.233$, which is taken from the *Cape Photographic Durchmusterung*. The class of spectrum and a brief description of the object are given in the sixth and seventh columns. Each of the new variables has been confirmed independently by Miss L. D. Wells, unless otherwise specified. Additional information regarding these objects is given in the remarks following the table. In the case of new variable stars, the right ascension is followed by the designation described in the *Annals*, **48**, 93, which gives the approximate position, and also the designation described in the *Annals*, **53**, No. 7, which indicates the number in the series of variables found at Harvard. This last number is also given in the table, for convenience of future reference.

A photograph of the variable star, *R Cygni*, taken with the 8-inch Draper telescope on November 19, 1890, shows the spectrum of this star as Class Md, having the hydrogen lines $H\gamma$ and $H\delta$ bright. On a photograph taken with the same instrument, on December 7, 1904, the spectrum

[1] *Harvard College Observatory Circular* No. 111.

of this star appears to be of the fourth type, closely resembling Class Na, and shows no trace of bright hydrogen lines.

TABLE I

PECULIAR SPECTRA

Constellation	B. D. No.	R.A. 1900	Dec. 1900	Mag.	Spectrum	Description
		h m	° ′			
Cassiopeia ...	.....	1 32.4	+57 39	.. ..	Pec.	Bright lines. Type V.
Horologium..	−59°233	2 45.1	−59 28	.....	Mc 5 d	Variable. H 1201.
Orion.....	+ 1°1005	5 19.6	+ 1 45	5.0	B Pec.	Hβ bright.
Auriga.....	R	5 48.9	+46 6	.....	Pec.	Brig't lines. Gaseous Neb.
Gemini.....	+21°1609	7 23.3	+21 7	.....	Pec.	Type IV.
Sextans.....	.....	10 29.9	+ 0 11	.....	.....	Variable. H 1202.
Crater.....	−6°3469	11 47.6	− 7 3	.....	Mc 5 d	Variable. H 1203.
Corona.....	.....	15 45.2	+36 35	.....	Md	Variable. H 1204.
Draco	+54°1925	17 55.6	+54 41	.....	Md	Variable. H 1205.
Sagitta.....	.....	19 15.0	+17 2	.....	Md?	Variable. H 1206.
Sagitta.....	.....	19 15.4	+17 5	.....	.....	Variable. H 1207.
Cygnus	....	19 23.3	+47 9	.....	Pec.	Bright lines. Type V.
Aquila.....	+12°4228	20 2.3	+12 39	.....	Md	Variable. H 1208.
Cygnus.....	+36°3907	20 5.8	+36 33	5.5	B Pec.	Hβ bright.
Cygnus.....	.....	20 14.7	+34 3	.....	Md	Variable. H 1209.
Cygnus.....	.....	20 24.7	+38 17	.....	Pec.	Bright lines. Type V.
Pegasus.....	R	22 1.5	+33 1	.....	.....	Variable. H 1210.
Pegasus.....	R	22 1.5	+33 1	.....	Md	Variable. H 1211.
Pegasus.....	+11°4784	22 16.6	+11 42	5.5	B Pec.	Hβ bright.
Lacerta.....	.....	22 41.8	+54 33	.....	Na	Type IV.
Lacerta.....	+53°3033	22 51.9	+53 41	9.1	Na	Type IV.
Cassiopeia ..	+59°2683	23 10.8	+59 55	9.0	Pec.	Bright lines. Type V.
Andromeda ..	+34°4974	23 33.8	+35 13	.....	Na	Type IV. Var. H 1212.
Andromeda ..	+42°4827	23 59.5	+43 0	.....	Na	Variable. H 1213.

REMARKS

h m

1 32.4. Galactic longitude, 96° 59′. Galactic latitude, −3° 38′.

2 45.1. *024559* = H 1201. An examination of this star on eight chart plates, taken between September 27, 1895, and December 2, 1902, shows a variation of about 1 3 magnitudes. Estimates from these plates give the approximate limits, 8.1 to 9.4.

5 19.6. *Hγ* is also visible as a fine bright line superposed on a faint dark band.

5 48.9. Galactic longitude, 133°. Galactic latitude, +12°. Assuming an error of +2′.8 in declination in the *Durchmusterung*, this object identifies as +46°.1067 magn. 9.5. The position given is that of the object on the photograph. No evidence of proper motion is shown from a comparison of plates taken on December 29, 1890, and January 16, 1901.

7 23.3. This object is in *N. G. C.* 2392, or identical with it. *N. G. C.* 2392 was found to have a continuous spectrum, with three bright lines, by Winlock and Peirce, on January 7, 1869. *Annals* **13**, 68. It was also found to be gaseous in 1872, by d'Arrest, *Astron. Nach.*, **79**, 193. Photographs taken with the 8-inch Draper telescope, on November 21, 1900, and November 27, 1905, show no trace of the bright lines characteristic of gaseous nebulæ, but that the spectrum resembles

the fourth type. The images on chart plates, however, are hazy as compared with those of adjacent stars, and the image on a photograph taken with the Bruce telescope on April 16, 1904, shows distinct nebulosity, especially on the preceding and southern edges.

10 29.9. 102900=H 1202. Discovered by Miss E. F. Leland, by superposition of chart plates, and confirmed by Mrs. Fleming. An examination of this star on five chart plates, taken between April 24, 1891, and February 11, 1905, shows a variation of about 1.6 magnitudes. Estimates from these plates give the approximate limits 8.9 to 10.5.

11 47.6. *114707*=H 1203. An examination of this star on twenty-two chart plates, taken between May 11, 1891, and March 6, 1905, shows a variation of about 1.0 magnitude. Estimates from these plates give the approximate limits, 8.2 to 9.2.

15 45.2. 154536=H 1204. An examination of this star on ten chart plates, taken between July 23, 1890, and May 21, 1903, shows a variation of about 3.0 magnitudes. Estimates from these plates give the approximate limits, 8.3 to 11.5.

17 55.6. 175554=H 1205. An examination of this star on forty-seven chart plates, taken between August 17, 1892, and July 12, 1905, shows a variation of at least 0.6 magnitude.

19 15.0. 191517a=H 1206. An examination of this star on ten chart plates, taken between August 19, 1896, and October 15, 1900, shows a variation of at least 2.8 magnitudes. Estimates from these plates give the approximate limits, 8.7 to <11.5.

19 15.4. 191517b=H 1207. This star was selected as comparison star *o* for 191517a. In observing the latter star, it was found by Miss S. E. Breslin to be brighter than the comparison star *n* on a plate taken on March 26, 1903. An examination of this star, by Mrs. Fleming, on ten chart plates, taken between May 1, 1900, and March 26, 1903, shows a variation of at least 0.7 magnitude. Estimates from these plates give the approximate limits 12.2 to 12.9.

19 23.3. Galactic longitude, 46° 38′. Galactic latitude, +13° 23′.

20 2.3. 200212=H 1208. An examination of this star on four chart plates, taken between September 18, 1892, and September 9, 1901, shows a variation of at least 1.7 magnitudes. Estimates from these plates give the approximate limits 9.5 to <11.2.

20 5.8. The hydrogen line $H\beta$ appears as a fine bright line centrally superposed on the dark line $H\beta$, on plates taken with the 11-inch Draper telescope on July 4 and November 4, 1905.

20 14.7. 201434=H 1209. An examination of this star on five chart plates, taken between September 9, 1893, and May 23, 1899, shows a variation of about 2.6 magnitudes. Estimates from these plates give the approximate limits, 9.9 to 12.5.

20 24.7. Galactic longitude, 45° 12′. Galactic latitude, −0° 51′.

22 1.5. 220133a=H 1210. The preceding and southern of two stars, 40″ apart, both of which are variable, and either of which might be identified as +32°.4335, magnitude 9.5. An examination of this star on sixteen chart plates, taken between December 22, 1890, and August 31, 1902, shows a variation of about 0.6 magnitude. Estimates from these plates give the approximate limits, 10.0 to 10.6.

22 1.5. 220133b=H 1211. See 220133a. An examination of this star on nine chart plates, taken between December 22, 1890, and November 23, 1900, shows a variation of about 2.4 magnitudes. Estimates from these plates give the approximate limits, 10.0 to 12.4.

22 51.9. Near the border of *Andromeda*.

23 10.8. Galactic longitude, 79° 2′. Galactic latitude, −0° 11′.

23 33.8. 233335=H 1212. An examination of this star on five charts plates, taken between September 24, 1890, and November 29, 1899, shows a variation of more than 2.3 magnitudes. Estimates from these plates give the approximate limits, 8.2 to<10.5.

23 59.5. 235943=H 1213. Suspected of variability by Espin. *Astron. Nach.*, **137**, 373, and *A. J.*, **16**, 171. Discovered here, independently, from its photographic spectrum. An examination of ten chart plates, taken between October 8, 1891, and November 25, 1903, shows a variation of about 1.5 magnitudes. Estimates from these plates give the approximate limits, 8.3 to 9.8.

SPECTRA OF KNOWN VARIABLES

The spectra of a number of known variables have also been determined from Draper photographs, and are given in Table II. This contains those classified since *Circular* 98 was issued, and therefore includes a few already mentioned in the Sixtieth Annual Report of the director. They are reprinted here for convenience of reference. The first column contains the designation, and the second, the name of the variable. The third column gives the class of spectrum.

TABLE II

SPECTRA OF KNOWN VARIABLES

Desig.	Name	Spectrum	Desig.	Name	Spectrum
004533	*R R Andromedae*	Md 543	190967	*U Draconis*	Md?
042215	*W Tauri*	Mb	191350	*TZ Cygni*	Mc
063558	*S Lyncis*	Md 8	194348	*TU Cygni*	Md 6
133273	*T Ursae Minoris*	Md 6	195849	*Z Cygni*	Md 6 Pec.
153378	*S Ursae Minoris*	Md 7	200647	*SV Cygni*	Na
161138	*W Coronae*	Md 4			

Desig.	Name	Spectrum
210812	*R Equulei*	Md 5
210903	*RR Aquarii*	Md 4
220412	*T Pegasi*	Md 9
220714	*RS Pegasi*	Md 9, 8, 8
233956	*Z Cassiopeiae*	Md 7

REMARKS

042215. The spectrum of this star is given as F? in the "Provisional Catalogue of Variable Stars," *Annals*, **48**, No. 3. Plate I, 33567, taken on October 25, 1905, shows the spectrum of this star as Class Mb.

153378. The spectrum of this star is given as Mc 5 d in the "Provisional Catalogue of Variable Stars," *Annals*, **48**, No. 3. Plate I, 33601, taken on November 1, 1905, shows the spectrum of this star as Class Md 7.

195849. The spectrum of this star is given as Mc? in the "Provisional Catalogue of Variable Stars," *Annals*, **48**. No. 3. Plate I, 33706, taken on November 27, 1905, shows the spectrum of this star as Class Md 6.

A photograph, A 6911, of the region whose center is in R. A. $=20^h$ 45^m, Dec. $=+30°.6$, was taken with the 24-inch Bruce telescope, on September 2, 1904, with an exposure of 240^m. It shows the entire region containing the nebulae *N.G.C.* 6960 and *N.G.C.* 6992. From an examination of this plate, Mrs. Fleming found that the northern and perhaps the southern ends of these nebulæ are connected by faint nebulosity forming an irregular oval. A large triangular wisp of nebulosity extends southward, from the north preceding portion of this oval, and is much more conspicuous than the neighboring nebulæ, *N.G.C.*6974 and *N.G.C.* 6979.

EDWARD C. PICKERING.

FEBRUARY 16, 1906.

THE SPECTRUM OF *NOVA AQUILAE* NO. 2

Nova Aquilae No. 2 was observed visually with the one-prism spectrograph on the night of September 5, when its magnitude was about 10.5 on the Harvard scale. At that time the spectrum was similar to that of *Nova Geminorum*, as observed by Drs. Reese and Curtis on April 1, 1903.[1] The spectrum consisted of a number of bands, the brightest of which was easily identified as H_β by means of the neighboring iron lines in the iron spark. A faint band in the region of $\lambda 4600$, and the still fainter H_γ band, could also be distinguished. A series of maxima extending from the region of H_β toward the red, giving almost the appearance of a continuous spectrum, was also observed. The seeing was poor and the image very faint, due to a great amount of smoke in the air, making the identification of the various bands (with the exception of H_β) quite difficult.

Although the *Nova* was very faint, three spectrograms of it were obtained with the one-prism spectrograph as follows.

[1] *L. O. Bulletin*, **2**, 59, 60; *Astrophysical Journal*, **18**, 299, 1903.

Plate	Date	Exposure
3986 A	1905, September 6	3 hours
3994 A	September 10	4 hours
4058 A	October 11	3½ hours

The plates of September 6 and 10, in the region common to the visual and photographic rays, confirm the observations of September 5. The exposure time was about right for the H_β band, and much too short for the others. The following is a brief description of the bands and their approximate wave-lengths, which were obtained by interpolation from the iron comparison lines.

H_β band: strong; limits, λ 4845–4885; edges fairly sharp.

Band at λ 4600: intensity about one-fifth that of H_β; limits not sharply defined, but approximately from λ 4590–4710; fades off gradually on both sides.

H_γ band: intensity one-tenth (or less) that of H_β; width 50–60 t.-m.; sharp minimum at λ 4345.

H_δ band: very faint; width about 70 tenth-meters.

A faint continuous spectrum extends from the region λ 4500 to that of the H_γ band. H_ϵ and the so-called nebular lines do not appear on the plates. It will be noticed that the relative photographic intensities given above are very unlike those of *Nova Geminorum* in April 1903. In the case of *Nova Aquilae No.* 2 the visual and photographic observations of the early part of September agree in showing that much of the light in the visible spectrum was concentrated in the H_β band. Visual observations of *Nova Geminorum* on April 1, 1903, gave H_β much brighter than the other bands, but the spectrograms of April 2 and following nights indicate a photographic intensity for H_β much less than that of the λ 4600 H_γ and H_δ bands. While this may be due to a real difference in the stars, some of it undoubtedly arises from the difference in sensitiveness between Cramer Crown (used by the former observers) and Seed 27 plates (used in the present case).

On account of other lines of work and very poor atmospheric conditions, it was impossible to obtain another observation of the *Nova* until October 11, at which time its magnitude was about 11.4 (Harvard scale). Visual observations on that evening showed that the H_β band had decreased very much in intensity since September 10, and was now only about as bright as the other faint bands. The spectrogram taken the same night while very badly underexposed shows also this marked decrease in intensity of the H_β band, which on this plate is as faint as the H_γ band.

The author desires to acknowledge his indebtedness to Mr. S. Albrecht for his efficient assistance in securing the above spectrograms.

J. H. MOORE.

LICK OBSERVATORY,
March 7, 1906.

A LIST OF FOUR STARS WHOSE RADIAL VELOCITIES ARE VARIABLE

The following stars have been found to have variable velocities in the line of sight:

τ Ursae Majoris ($\alpha = 9^h\ 2^m.7$; $\delta = +63°\ 55'$)

Plate	Date	Velocity	Measured by
1626 D	1900 January 22	− 5	Campbell
		− 3.8	Stebbins
2377 A	1902 April 15	−10	Reese
		−10.2	Stebbins
2624 E	1902 December 30	− 4	Curtis
		− 6.0	Burns
3107 F	1903 December 25	− 9.7	Moore
4181 E	1906 January 29	− 1	Moore
		− 1.5	Burns

The character of spectrum is given by Harvard as of type XII *c*. The variable velocity was suspected from the second plate and confirmed by the recent measures of Messrs. Moore and Burns.

λ *Hydrae* ($\alpha = 10^h\ 5^m.7$; $\delta = -11°\ 51'$)

Plate	Date	Velocity	Measured by
686 A	1898 March 30	+23.3	Wright
1174 D	1899 February 13	+22.9	"
1648 C	1900 February 2	+18.6	"
1660 A	February 26	+19.4	"
1671 D	March 9	+18.4	"
1675 B	March 12	+19.2	"
1682 C	March 13	+19.0	"
1694 C	March 27	+19.2	"
1977 D	December 5	+15.1	Burns
2010 D	1901 January 15	+17.4	Reese
2321 D	December 23	+21.4	"
2627 C	1902 December 31	+24.1	Burns
2706 D	1903 February 23	+22.5	"
3187 A	1904 March 31	+19.9	Brasch
		+18	Moore

The spectrum is of type K. The variation was suspected by Mr. W. H. Wright from the observations of 1898, 1899, and 1900, and confirmed by the recent measures of Mr. K. Burns.

μ Ursae Majoris ($\alpha = 10^h\ 16^m.4$; $\delta = +42°\ 0'$)

Plate	Date	Velocity	Measured by
309 A	1897 February 24	−24	Campbell
		−27.4	Burns
689 A	1898 March 31	−20	Campbell
		−22.0	Burns
1201 C	1899 March 6	−20	Campbell
		−19.1	Burns
3208 D	1904 April 11	−16	Moore
		−16.2	Burns
4151 D	1906 January 4	−23	Moore

The spectrum is of type M.

The binary character of the above three stars was discovered with the Mills spectrograph. Unfortunately the plates are not properly distributed to give an idea of the period of any of them.

Y Ophiuchi ($\alpha = 17^h\ 47^m.3$; $\delta = -6°\ 07'$)

This star was discovered to be a spectroscopic binary by Mr. S. Albrecht (Fellow in the Observatory) from a series of observations made with the one-prism spectrograph during the summer and fall of 1905. The period coincides with the period of light-variation, which is 17.12 days. While this star has already been announced[1] by the discoverer, it is included here for completeness.

J. H. MOORE.

MARCH 7, 1906.

SPECTROGRAPHIC OBSERVATIONS

FOUR STARS WITH VARIABLE RADIAL VELOCITIES

B. D. −1°.1004 ($\alpha = 5^h\ 36^m$; $\delta = -1°\ 11'$; Mag. = 5.1)

Plate	Date	G. M. T.	No. Lines Measured	Velocity
I B 518	1905 February 13	$15^h\ 31^m$	5	+ 30 km
I B² 659	1906 January 26	15 56	8	+ 19
683	February 12	13 53	6	+132
688	February 16	12 55	6	− 34

The spectrum is of the *Orion* type, and the lines are broad, least so on Plate 688. The helium line λ 4388, although about the best measurable line, appears to have a distinctly different displacement from the other lines on most of the plates, and it is not used in deriving the velocities. There does not seem to be other evidence of the visible existence of a

[1] *Publications of the A. S. P.*, **18**, 66, 1906.

second component spectrum. The last two observations show that the period is short and the range large.

29 Canis Majoris ($\alpha = 7^h\ 14^m$; $\delta = -24°\ 23'$; Mag. = 4.8)

Plate	Date	G. M. T.	No. Lines Measured	Velocity
I B² 661.....	1906 January 26	$17^h\ 49^m$	2	−164 km
675.....	January 29	17 52	3	− 3
685.....	February 12	15 40	4	−243
690.....	February 16	14 56	4	− 92

The first and third plates were under-exposed, and the results are uncertain by many kilometers, but the displacements are remarkably large. The period is evidently short. The determinations of velocity depend chiefly upon $H\gamma$ and *He* λ4472; but on the last two plates $H\beta$ and λ4542 were also used.

In Harvard *Circulars* Nos. 16, 17, and 32, Professor E. C. Pickering called attention to the presence in this spectrum of the lines characteristic of *ζ Puppis*. It is made by Miss Cannon[1] the typical star of Class Oe, also showing a number of lines in addition to those seen in *ζ Puppis*.

Although I have had this star on the observing program of the Bruce spectrograph for several years, it was not until this season that time could be found for obtaining its spectrogram. The lines are so broad and diffuse on the four plates I have secured that an accurate determination of the wave-lengths of the lines of the *ζ Puppis* series unfortunately will not be possible.[2]

In this connection I may mention that on two nights of this winter I have been able to get sufficiently exposed spectrograms of *ζ Puppis* itself, although its meridian altitude is less than 8°. Here, again, the lines are so broad with the slit-spectrograph as to preclude accurate measurements. There are, however, other stars, notably *10 Lacertae*, in the spectra of which the "additional hydrogen lines" appear much more sharp than in the prototype. We have obtained numerous plates of this star which will be utilized, as soon as time permits, for the accurate determination of the wave-lengths of such of these lines as are in the region of good focus on our plates. Our plates of λ *Cephei*, recently noted by Professor Pickering as exhibiting these lines, will not yield results of precision on account of the diffuseness of all the lines in the spectrum.

[1] Harvard *Annals*, **28**, 148–50.

[2] Note added to proofsheets. Spectrograms just obtained of the star *30* or *τ Canis Majoris* (*24'* south of *29*), which has a somewhat similar spectrum, show that it also has a variable radial velocity of large range.

μ Orionis ($\alpha = 5^h\ 57^m$, $\delta = +9°\ 39'$, Mag. = 4.3)

Plate	Date	G. M. T.	No. Lines Measured	Velocity
II B 25	1905 November 24	$18^h\ 53^m$	13	+65 km
I B 624	December 9	18 6	6	+46
I B^2 650	December 25	16 32	7	+58
B^2 623	1906 January 5	16 55	16	+37.8
B^2 631	January 8	18 58	13	+71.5
I B^2 669	January 29	14 12	5	+21
B^2 641	February 9	15 0	18	+63.6

This spectrum is of type Ia2–Ia3, not differing much from that of *α Cygni*, and is well suited for measurement. The observations of January 5 and 8 indicate that the period is short. A rather high velocity of the system is suggested, though it is quite likely that smaller values of the radial velocity will be found on future plates than on those so far obtained.

Some of the plates give faint evidences of the lines of the spectrum of a second component. Thus three such faint lines were measured on plate B^2623, and gave accordant values, the mean of which was +104 km. I am not yet prepared, however, to vouch for the visible reality of the second component. The plates taken with one prism (I B), and two prisms (II B), are naturally less accurately measurable than those obtained with the full dispersion of three prisms (B).

T Monocerotis ($\alpha = 6^h\ 20^m$; $\delta = +7°\ 8'$; Mag. = 6–8)

Plate	Date	G. M. T.	No. Lines Measured	Velocity
I A 9	1905 November 3	$21^h\ 30^m$	11	+17 km
II B 26	November 24	21 40	14	+13
I B 643	December 15	20 30	14	+ 7

This variable star has a spectrum of the solar type, and can be well measured if high enough dispersion is employed. The star's faintness makes a long exposure necessary and prevents the accumulation of plates, particularly as the star is situated near many others of our observing programs which also require attention. The exposure times for the three plates above were 210, 224, and 140 minutes, respectively. The result for the first plate is least reliable; that for the second, the most so. The range of velocity so far observed is rather small, perhaps less than might be expected; but I believe it to be real. The star's period is 27.0 days, the maximum occurring 8 days after minimum. According to the current ephemeris, the first plate was taken 19 days after minimum, or 8 days

before maximum; the second plate at five days after maximum, and the third plate at one day before maximum. It seems hardly worth while, however, to draw any inferences from these relations until additional plates are available.

The plates so far referred to in this note were taken by the writer, with the assistance of Mr. Sullivan.

OBSERVATIONS OF *a DRACONIS*

This star I casually placed on the observing list for the latter part of the night of January 5. A glance at the plate next morning showed a large negative displacement (subsequently measured to be —50 km, unreduced to the Sun). Visual comparison of this plate with the only one obtained up to that time with the Bruce spectrograph, taken by Mr. Adams on November 20, 1901, which had a positive displacement, indicated that the star's velocity varies. I carelessly overlooked the previous announcement of this fact by Messrs. Campbell and Curtis[1] until several new plates had been obtained and measured.

Meanwhile appeared Mr. Moore's interesting note[2] on the Lick observations of this star, which by a remarkable coincidence yielded on five dates a practically identical result (—40 to —43 km), with a different value (0 km) on only one date, upon which the establishment of the binary character of the star depended. The communication of our results, therefore, would seem to be appropriate at this time. All the plates have been taken by Mr. Barrett, with the exception of the first, obtained by Mr. Adams.

Plate	Date	G. M. T.	No. Lines Measured	Velocity
B 245	1901 November 20	$22^h\ 8^m$	8	+20 km
B^2 627	1906 January 5	23 37	8	−42
B^2 633	January 8	21 56	1	−55
I B^2 664	January 26	21 17	5	− 9
I B^2 678	January 29	19 28	3	+ 1
B^2 644	February 9	22 20	7	+24

The first plate was measured in 1902 by Mr. Adams, who obtained from 6 lines a value agreeing within half a kilometer with my recent measure given above.

The value for January 8 depends upon the excellent *Mg* line λ4481. The above observations appear to be satisfied by a period of between 51 and 52 days, which apparently also fits the Lick observations.

[1] *Astrophysical Journal*, **18**, 307, 1903.

[2] *Publications of the A. S. P.*, **18**, 66, 1906.

The spectrum is assigned by Vogel and Wilsing to type Ia2, and by Miss Maury to group VII*a*. The helium lines of the *Orion* type are faintly visible on some of the plates, and I was able to measure λ4472 on one plate, while on one of the one-prism plates I measured the silicon lines at λ4128 and 4131. Some of the enhanced lines of *Ti* and *Fe* are quite well measurable. The star should strictly be assigned to the *Orion* type, a remark which would apply to some other stars ordinarily classified under Ia2. (For instance Miss Maury[1] notes a trace of λ4472 in the spectrum of α *Lyrae*, which appears to be confirmed by our plates.)

SPECTRUM OF *PLEIONE*

In recently photographing the spectrum of *Pleione*, I have been surprised by the faintness or absence of the bright hydrogen lines, which were fairly conspicuous on spectrograms obtained elsewhere a few years ago. Their presence (superposed upon the broader dark lines) was first announced by Professor E. C. Pickering,[2] as a result of an examination of the Harvard plates by Miss Maury.

I recall taking several spectrograms of the star at Potsdam in 1891 or 1892, which plainly showed the emission lines. Our plates were taken on the following dates: 1905, November 10, December 4, December 25; 1906, January 26, January 29, February 19.

Professor Pickering has kindly informed me in regard to the plates taken with the objective-prism. Bright $H\beta$ was well shown on negatives taken December 28, 1888, and December 31, 1896. An examination of twenty-two plates taken on thirteen nights in the autumn of 1896 has been made by Miss Cannon, and she is unable to detect any certain changes in the intensity of bright $H\beta$ on these plates.

I am permitted to quote from a letter dated February 20, 1906, in which Professor Pickering says: "The line $H\beta$ does not now appear bright on our photographs of *Pleione*." It does not appear on a print of the *Pleiades* from a negative taken January 30, 1906, with the same apparatus as the early photographs, which illustrates Harvard *Circular* No. 110. (See p. 256 of this issue.) It would be interesting to know what testimony may be given by plates taken at other observatories in regard to variability of the intensity of the bright lines of *Pleione*.

The equipment of the Bruce spectrograph has recently been much improved by the addition of a triple uncemented camera lens, designated above as B[2], which was designed by Professor Hastings and constructed by Brashear. Its aperture is 57 mm and its focal length is 608 mm.

[1] Harvard *Annals*, **28**, 23.

[2] *Astronomische Nachrichten*, **123**, 95, 1889.

It has now been in use for nearly three months, and has been entirely free from the capricious changes in definition which have caused us great annoyance with our previous cemented triplets. It has not been found necessary to increase our exposure times appreciably as compared with those given with the cemented camera lenses.

EDWIN B. FROST.

YERKES OBSERVATORY,
March 14, 1906.

NOTE ON THE D. O. MILLS EXPEDITION TO THE SOUTHERN HEMISPHERE.

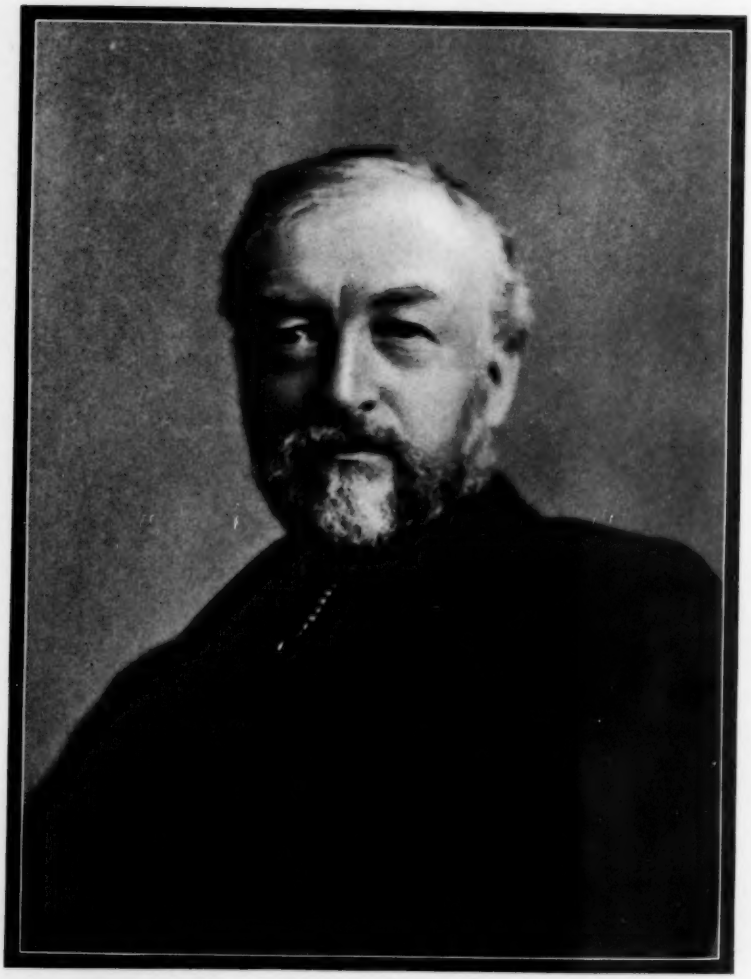
The two-year period of actual observation originally planned for the D. O. Mills Expedition to Santiago, Chile, terminated in October 1905, the installation of the observatory having been completed in October 1903. As has already been announced in *Publications of the Astronomical Society of the Pacific*, Mr. Mills has most generously offered to continue the work of the expedition for five additional years. Acting Astronomer Heber D. Curtis, accompanied by his family, sailed from San Francisco on December 30, 1905, via Panama, to Chile, to take charge of the observatory for this new period of work. He expected to reach his destination about February 15. Immediately following Dr. Curtis' arrival, Acting Astronomer W. H. Wright and Mrs. Wright will return to Mount Hamilton. As Dr. Curtis' duties in connection with the Crocker Eclipse Expedition to Labrador prevented him from leaving earlier for Chile, Professor Wright has kindly extended the term of his residence there, awaiting the arrival of his successor.

Dr. H. K. Palmer, assistant in the D. O. Mills Observatory, returned to Mount Hamilton in December 1905, after an absence of nearly three years. He is engaged at present in measuring and reducing the spectrograms secured in Chile. His successor will soon leave for Santiago. Mr. Mills has at the same time provided for extensive improvements in and additions to the equipment of the observatory, several of which are nearing completion. The 30-foot Warner and Swasey dome of steel frame, formerly covered with heavy painted canvas, has been re-covered with metal. A ball thrust-bearing and two roller side-bearings have been provided for the declination axis of the telescope. Two-prism and one-prism spectrographs have been built, in order that radial velocity determinations may be carried to fainter stars. Professor Wright's experience has made it practically certain that rapid changes in focal length and other sources of disturbance in the stellar images are due to rapid changes

of the temperature of the mirror during the first hours of the night. The question of maintaining artificially the temperature of large mirrors during the daytime at the reading estimated for the atmosphere for the evening that follows has often been discussed in past years by the members of our staff and by others. During my absence in Europe last summer and fall Dr. Curtis worked out the details for such a refrigerating scheme, and the apparatus has been shipped for installation and trial on Cerro San Cristobal. Apparatus for quick re-silvering of the 37-inch mirror is under construction. Another building will be erected on the summit of the mountain to accommodate a machine shop, equipped with lathe and small tools, and to contain rooms for the observers. An electric power line will be installed to convey current from the city for the machine shop, refrigerator, and comparison arc, in addition to minor uses. Telephone connections between the observatory on the summit and the astronomer's residence in the city, and many other lesser conveniences and improvements will be made.

W. W. CAMPBELL.

FEBRUARY 25, 1906.



SAMUEL PIERPONT LANGLEY